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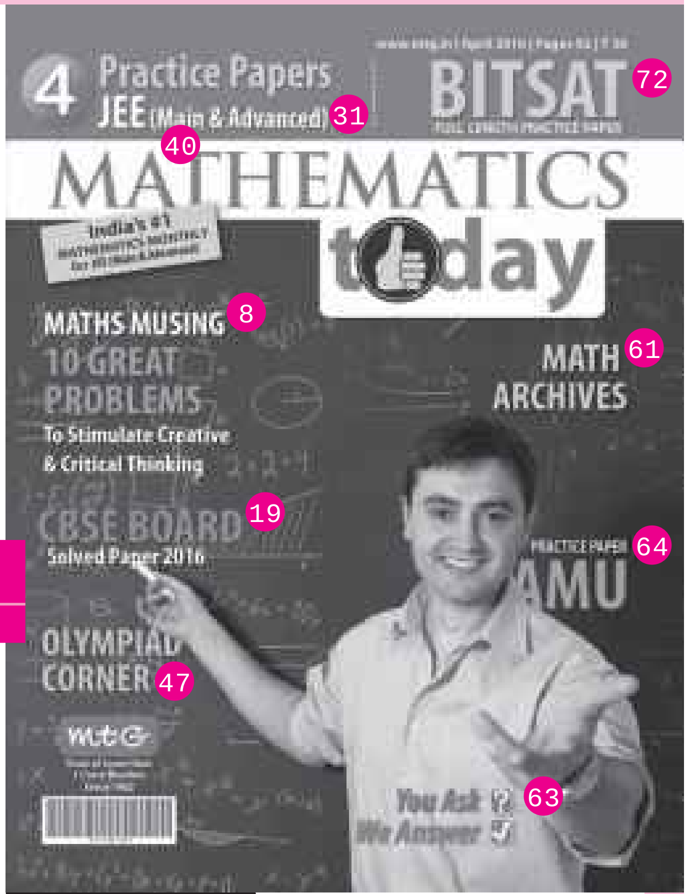

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MATHS MUSING

Maths Musing was started in January 2003 issue of Mathematics Today with the suggestion of Shri Mahabir Singh. The aim of Maths Musing is to augment the chances of bright students seeking admission into IITs with additional study material.

During the last 10 years there have been several changes in JEE pattern. To suit these changes Maths Musing also adopted the new pattern by changing the style of problems. Some of the Maths Musing problems have been adapted in JEE benefitting thousand of our readers. It is heartening that we receive solutions of Maths Musing problems from all over India.

Maths Musing has been receiving tremendous response from candidates preparing for JEE and teachers coaching them. We do hope that students will continue to use Maths Musing to boost up their ranks in JEE Main and Advanced.

PROBLEM Set 160

JEE MAIN

1. If S is the sum of all the digits of the natural number $N = 1 + 11 + 111 + 1111 + \ldots\ldots$ to 2011 terms, then the sum of the digits of S is

(a) 17 (b) 18 (c) 19 (d) 21

2. If a, b, c are complex numbers such that $|a| = 1$, $|b| = \sqrt{2}, |c| = \sqrt{3}$ and $|a + b + c| = 2$, then $|bc + 2ca + 3ab| =$

(a) $\sqrt{6}$ (b) $2\sqrt{2}$ (c) $2\sqrt{3}$ (d) $2\sqrt{6}$

3. $\sec^4\theta - 8\sec^2\theta\tan\theta + 16\tan^2\theta = 0$ if $\theta =$

(a) $\frac{\pi}{12}$ (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{3}$ (d) $\frac{5\pi}{12}$

4. A plane passes through the points (6, –4, 3) and (0, 4, –3). The sum of its intercepts on the coordinate axes is zero. Its distance from the origin is

(a) $\frac{3}{7}$ (b) $\frac{6}{7}$ (c) $\frac{12}{7}$ (d) $\frac{18}{7}$

5. A man goes in for an examination in which there are four papers with a maximum of 20 marks. If N is the number of ways of getting 40 marks on the whole, then sum of the digits of N is

(a) 14 (b) 15 (c) 16 (d) 18

JEE ADVANCED

6. If $\left(\frac{1}{2}\right)^{\log_3 \log_{1/5}\left(x^2-\frac{4}{5}\right)} > 1$ then $|x| \in$

(a) $\left(\frac{2}{\sqrt{5}}, 1\right)$ (b) $\left(\frac{2}{\sqrt{5}}, \frac{3}{\sqrt{5}}\right)$

(c) $\left(1, \frac{3}{\sqrt{5}}\right)$ (d) $(1, \infty)$

COMPREHENSION

A fair die is rolled four times. The probability

7. that each of the final three rolls is atleast as large as the roll preceding it is

(a) $\frac{5}{18}$ (b) $\frac{5}{9}$

(c) $\frac{7}{36}$ (d) $\frac{7}{72}$

8. that the list of outcomes contains exactly 3 distinct numbers is

(a) $\frac{5}{18}$ (b) $\frac{5}{9}$

(c) $\frac{7}{36}$ (d) $\frac{7}{72}$

INTEGER MATCH

9. If $a + b + c = 0$, $a^3 + b^3 + c^3 = 3$ and $a^5 + b^5 + c^5 = 10$, then $a^4 + b^4 + c^4$ is

MATRIX MATCH

10.

	Column I		Column II
(a)	If $\binom{n}{r-1} = 165, \binom{n}{r} = 330, \binom{n}{r+1} = 462$, then n is	(p)	8
(b)	If the equation $x^2 + x - n = 0$ has integer roots, the number of values of n between 1 and 100 is	(q)	9
(c)	A circle is described on any focal chord of $y^2 = 20x$ as diameter. The locus of its centre is a conic of latusrectum	(r)	10
		(s)	11

See Solution set of Maths Musing 159 on page no. 30

PRACTICE PAPER 2016

JEE ADVANCED

PAPER-1

SECTION - I

INTEGER ANSWER TYPE

1. In the expansion of $(3^{-x/4} + 3^{5x/4})^n$, the sum of the binomial coefficients is 64 and the term with the greatest binomial coefficient exceeds the third by $(n - 1)$ then find the value of x.

2. Let $p(x)$ be a polynomial of degree 4 having extremum at $x = 1, 2$ and $\lim\limits_{x\to 0}\left(1+\dfrac{p(x)}{x^2}\right) = 2$. Then the value of $p(2)$ is

3. The value of $\dfrac{1}{81^n} - \dfrac{10}{81^n}\,{}^{2n}C_1 + \dfrac{10^2}{81^n}\,{}^{2n}C_2 - \dfrac{10^3}{81^n}\,{}^{2n}C_3 + ... + \dfrac{10^{2n}}{81^n}$ is

4. Consider an equilateral triangle having vertices at points $A\left(\dfrac{2}{\sqrt{3}}e^{\frac{i\pi}{2}}\right)$, $B\left(\dfrac{2}{\sqrt{3}}e^{\frac{-i\pi}{6}}\right)$ and $C\left(\dfrac{2}{\sqrt{3}}e^{\frac{-5\pi}{6}}\right)$. If $P(z)$ is any point on its incircle, then $AP^2 + BP^2 + CP^2 =$

5. Let $f(\theta) = \sin\left(\tan^{-1}\left(\dfrac{\sin\theta}{\sqrt{\cos 2\theta}}\right)\right)$, where $-\dfrac{\pi}{4} < \theta < \dfrac{\pi}{4}$. Then the value of $\dfrac{d}{d(\tan\theta)}(f(\theta))$ is

6. If $[x]$ stands for the greatest integer function, then the value of $\displaystyle\int_2^8 \frac{[x^2]\,dx}{[x^2 - 20x + 100] + [x^2]}$ is

7. The maximum value of the function $f(x) = 2x^3 - 15x^2 + 36x - 48$ on the set $A = \{x \mid x^2 + 20 \le 9x\}$ is

8. If $\sin^{-1} x \in \left(0, \dfrac{\pi}{2}\right)$, then find the value of
$$\tan\left[\frac{\cos^{-1}(\sin(\cos^{-1}x)) + \sin^{-1}(\cos(\sin^{-1}x))}{2}\right]$$

SECTION - II

ONE OR MORE THAN ONE CORRECT ANSWER TYPE

9. A man wants to divide 101 coins, a rupee each, among his 3 sons with the condition that no one receives more money than the combined total of other two. The number of ways of doing this is :

(a) ${}^{103}C_2 - 3\,{}^{52}C_2$ (b) $\dfrac{{}^{103}C_2}{3}$

(c) 1275 (d) $\dfrac{{}^{103}C_2}{6}$

10. The solution of the differential equation, $x(x^2 + 3y^2)dx + y(y^2 + 3x^2)dy = 0$ is

(a) $x^4 + y^4 + x^2y^2 = c$ (b) $x^4 + y^4 + 3x^2y^2 = c$

(c) $x^4 + y^4 + 6x^2y^2 = c$ (d) $x^4 + y^4 + 9x^2y^2 = c$

11. Let $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$ and $10B = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{bmatrix}$

If B is the inverse of the matrix A then α is

(a) – 2 (b) – 1 (c) 2 (d) 5

12. Let the eccentricity of the hyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$ be reciprocal to that of the ellipse $x^2 + 4y^2 = 4$. If the hyperbola passes through a focus of the ellipse, then

(a) the equation of the hyperbola is $\dfrac{x^2}{3} - \dfrac{y^2}{2} = 1$

(b) a focus of the hyperbola is (2, 0)

(c) the eccentricity of the hyperbola is $\sqrt{\dfrac{5}{3}}$

(d) the equation of the hyperbola is $x^2 - 3y^2 = 3$

13. Equation of the circle of radius 5 which touches x-axis and the line $3x = 4y$ is

(a) $x^2 + y^2 - 30x - 10y + 225 = 0$
(b) $x^2 + y^2 + 30x + 10y + 225 = 0$
(c) $x^2 + y^2 + (10/3)x - 10y + 25/9 = 0$
(d) $x^2 + y^2 - (10/3)x + 10y + 25/9 = 0$

14. If $x^y = y^x$; $x, y > 0$, then dy/dx is

(a) $\dfrac{y(x\log y - y)}{x(y\log x - x)}$ (b) $\dfrac{y^2(\log x - 1)}{x^2(\log y - 1)}$

(c) $\dfrac{yx^{y-1} - y^x \log y}{xy^{x-1} - x^y \log x}$ (d) none of these

15. If the derivative of an odd cubic polynomial vanishes at two different values of 'x' then

(a) coefficient of x^3 & x in the polynomial must be same
(b) coefficient of x^3 & x in the polynomial must be of different sign
(c) the values of 'x' where derivative vanishes are closer to origin as compared to the respective roots on either side of origin.
(d) the values of 'x' where derivative vanishes are far from origin as compared to the respective roots on either side of origin.

16. If $f(x) = \begin{cases} x^2 + 2, & x < 0 \\ 3, & x = 0 \\ x + 2, & x > 0 \end{cases}$, then

(a) $f(x)$ has a maximum at $x = 0$
(b) $f(x)$ is strictly decreasing on the left of 0
(c) $f'(x)$ is strictly increasing on the left of 0
(d) $f'(x)$ is strictly increasing on the right of 0

17. Number of real roots of the equation $\cos^7 x + \sin^4 x = 1$ in the interval $(-\pi, \pi)$ is less than

(a) 3 (b) 4 (c) 5 (d) 6

18. If θ be the angle subtended at $P(x_1, y_1)$ by the circle, $S \equiv x^2 + y^2 + 2gx + 2fy + c = 0$, then

(a) $\cot\theta = \dfrac{2\sqrt{S_1}}{\sqrt{g^2 + f^2 - c}}$

(b) $\theta = 2\cot^{-1}\left[\dfrac{\sqrt{S_1}}{\sqrt{g^2 + f^2 - c}}\right]$

(c) $\tan(\theta/2) = \dfrac{\sqrt{g^2 + f^2 - c}}{\sqrt{S_1}}$

(d) none of these

SECTION - III

MATRIX MATCH TYPE

19. Match the columns.

	Column I		Column II
(A)	$\int_{3.3}^{10.8} [x]dx$ equals	(P)	1/4
(B)	The point of maxima of $2^{x^{25}(1-x)^{75}}$ in $[0,1]$ is	(Q)	52
(C)	$\lim_{x\to 0} \dfrac{\sin[x]}{[x]}$ equals	(R)	1
(D)	$\int_0^{4\pi} \lvert\cos x\rvert dx$ equals	(S)	8
		(T)	0

20. Match the columns.

	Column I		Column II
(A)	A line from the origin meets the lines $\dfrac{x-2}{1} = \dfrac{y-1}{-2} = \dfrac{z+1}{1}$ and $\dfrac{x-(8/3)}{2} = \dfrac{y+3}{-1} = \dfrac{z-1}{1}$ at P and Q respectively. If length $PQ = d$, then d^2 is	(P)	– 4
(B)	The values of x satisfying $\tan^{-1}(x+3) - \tan^{-1}(x-3) = \sin^{-1}\left(\dfrac{3}{5}\right)$ are	(Q)	0
(C)	Non-zero vectors $\vec{a}, \vec{b}$ and $\vec{c}$ satisfy $\vec{a}\cdot\vec{b} = 0$, $(\vec{b}-\vec{a})\cdot(\vec{b}+\vec{c}) = 0$ and $2\lvert\vec{b}+\vec{c}\rvert = \lvert\vec{b}-\vec{a}\rvert$. If $\vec{a} = \mu\vec{b} + 4\vec{c}$, then the possible values of μ are	(R)	4
(D)	Let f be the function on $[-\pi, \pi]$ given by $f(0) = 9$ and $f(x) = \sin\left(\dfrac{9x}{2}\right)\Big/\sin\left(\dfrac{x}{2}\right)$ for $x \neq 0$. The value of $\dfrac{2}{\pi}\int_{-\pi}^{\pi} f(x)dx$ is	(S)	5
		(T)	6

PAPER-2

SECTION - I

INTEGER ANSWER TYPE

1. The value of $c + 2$ for which the area of the figure bounded by the curve $y = 8x^2 - x^5$; the straight lines $x = 1$ and $x = c$ and x-axis is equal to $\frac{16}{3}$, is

2. The number of solutions of $\sin^{-1}\left(\frac{1+x^2}{2x}\right) = \frac{\pi}{2}\sec(x-1)$ is

3. If $|z_1| = 2, |z_2| = 3, |z_3| = 4$ and $|2z_1 + 3z_2 + 4z_3| = 4$ then the expression $|8z_2z_3 + 27z_3z_1 + 64z_1z_2|$ equals $24 \times k$. Find k

4. If the normals at the end points of a variable chord PQ of the parabola $y^2 - 4y - 2x = 0$ are perpendicular, then the tangents at P and Q will intersect at $mx + n = 0$. Find $m + n$.

5. If from a pack of 52 playing cards, one card is drawn at random, the probability that it is either a king or a queen is $\frac{k}{13}$. Find k.

6. Consider the set of eight vectors $V = \{a\hat{i} + b\hat{j} + c\hat{k} : a, b, c \in \{-1, 1\}\}$. Three non-coplanar vectors can be chosen from V in 2^p ways. Then p is

7. The value of
$$6 + \log_{\frac{3}{2}}\left(\frac{1}{3\sqrt{2}}\sqrt{4 - \frac{1}{3\sqrt{2}}\sqrt{4 - \frac{1}{3\sqrt{2}}\sqrt{4 - \frac{1}{3\sqrt{2}}\cdots\cdots}}}\right) \text{ is}$$

8. Let $f : R \to R$ and $g : R \to R$ be respectively given by $f(x) = |x| + 1$ and $g(x) = x^2 + 1$. Define $h : R \to R$ by
$$h(x) = \begin{cases} \max\{f(x), g(x)\}, & \text{if } x \le 0 \\ \min\{f(x), g(x)\}, & \text{if } x > 0 \end{cases}$$
The number of points at which $h(x)$ is not differentiable is

SECTION - II

ONE OR MORE THAN ONE CORRECT ANSWER TYPE

9. Let $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}, \vec{b} = \hat{i} + 2\hat{j} - \hat{k}, \vec{c} = \hat{i} + \hat{j} - 2\hat{k}$ be three vectors. A vector in the plane of $\vec{b}$ and $\vec{c}$ whose projection on $\vec{a}$ is of magnitude $\sqrt{2/3}$ is
   (a) $2\hat{i} + 3\hat{j} - 3\hat{k}$ (b) $2\hat{i} + 3\hat{j} + 3\hat{k}$
   (c) $-2\hat{i} - \hat{j} + 5\hat{k}$ (d) $2\hat{i} + \hat{j} + 5\hat{k}$

10. Which of the following is equivalent or connected with $f(x)$?
   (a) $\begin{cases} -1 & \text{if } x > 0 \\ 0 & \text{if } x = 0 \\ 1 & \text{if } x < 0 \end{cases}$ (b) $\begin{cases} \frac{x}{|x|} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$
   (c) $\begin{cases} \frac{|x|}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$ (d) $\begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{if } x = 0 \\ -1 & \text{if } x < 0 \end{cases}$

11. Let $f : R \to \left(0, \frac{\pi}{2}\right]$ be a function defined by $f(x) = \cot^{-1}(x^2 + 4x + \alpha^2 - \alpha)$, then complete set of values of α for which $f(x)$ is onto, is
   (a) $\left[\frac{1-\sqrt{17}}{2}, \frac{1+\sqrt{17}}{2}\right]$ (b) $\frac{1+\sqrt{17}}{2}$
   (c) $\left(-\infty, \frac{1-\sqrt{17}}{2}\right] \cup \left[\frac{1+\sqrt{17}}{2}, \infty\right]$
   (d) $\frac{1-\sqrt{17}}{2}$

12. Let $f(x) = \sin\frac{\{x\}}{a} + \cos\frac{\{x\}}{a}$ where $a > 0$ and $\{\cdot\}$ denotes the fractional part of function. Then the set of values of a for which f can attain its maximum values is
   (a) $\left(0, \frac{4}{\pi}\right)$ (b) $\left(0, \frac{2}{\pi}\right]$
   (c) $(0, \infty)$ (d) none of these.

13. The value of ${}^{2n}C_0 + {}^{2n}C_1 + \ldots + {}^{2n}C_n$ is
   (a) $2^{2n-1} + \frac{(2n-1)!}{n!(n-1)!}$ (b) $2^{n+1} + \frac{(2n+1)!}{(n+1)!(n-1)!}$
   (c) $2^n + \frac{(2n)!}{n!(n-1)!}$ (d) $2^{n+2} + \frac{(2n-1)!}{n!(n-1)!}$

14. Let $S_n = \sum_{k=1}^{4n} (-1)^{\frac{k(k+1)}{2}} k^2$. Then S_n can take value(s)
   (a) 1056 (b) 1088 (c) 1120 (d) 1332

15. If the straight line $\frac{x-1}{2} = \frac{y+1}{k} = \frac{z}{2}$ and $\frac{x+1}{5} = \frac{y+1}{2} = \frac{z}{k}$ are coplanar, then the plane(s) containing these two lines is (are)
   (a) $y + 2z = -1$ (b) $y + z = -1$
   (c) $y - z = -1$ (d) $y - 2z = -1$

16. Let $w = \frac{\sqrt{3}+i}{2}$ and $P = \{w^n : n = 1, 2, 3, ...\}$. Further $H_1 = \left\{z \in C : \text{Re } z > \frac{1}{2}\right\}$ and $H_2 = \left\{z \in C : \text{Re } z < -\frac{1}{2}\right\}$, where C is the set of all complex numbers. If $z_1 \in P \cap H_1$, $z_2 \in P \cap H_2$ and O represents the origin, then $\angle z_1 O z_2 =$

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{6}$ (c) $\frac{2\pi}{3}$ (d) $\frac{5\pi}{6}$

SECTION - III

COMPREHENSION TYPE

Paragraph for Question No. 17 and 18

If u and v are two function of x, then

$$\int uv\, dx = u\int v dx - \int\left\{\frac{du}{dx}\int v\, dx\right\}dx.$$

In applying the given rule, care has to be taken in the selection of first function and the second function. Normally if both of the functions are directly integrable then the first function is chosen in such a way that the derivative of the function thus obtained under integral sign is easily integrable. Now integrate the following.

17. $\int x \cos x\, dx =$

(a) $x \sin x + \sin x + C$ (b) $x \sin x + \cos x + C$
(c) $x \cos x + \sin x + C$ (d) none of these

18. $\int \log_e |x|\, dx =$

(a) $\log |x| - x + C$ (b) $x \log |x| + C$
(c) $x \log |x| - x + C$ (d) none of these

Paragraph for Question No. 19 and 20

The solution of differential equation is a relation between the variables of the equation not containing the derivatives, but satisfying the given differential equation. If y_1 and y_2 are two solutions of the differential equation $\frac{dy}{dx} + P(x)\cdot y = Q(x)$

19. General solution of differential equation is

(a) $y = y_1 x$ (b) $y = y_1 + c(y_1 - y_2)$
(c) $y_1 = y + cy_2$ (d) none of these

20. $\alpha y_1 + \beta y_2$ will also be a solution if

(a) $\alpha + \beta = 1$ (b) $\alpha + \beta = -1$
(c) $\alpha + \beta = 0$ (d) $\alpha + \beta = 2$

SOLUTIONS

PAPER-1

1. **(0) :** Given sum of the binomial coefficients in the expansion of $(3^{-x/4} + 3^{5x/4})^n = 64$

Then putting $3^{-x/4} = 3^{5x/4} = 1$

$\therefore \quad (1 + 1)^n = 64 \quad \Rightarrow 2^n = 64$

$\therefore \quad n = 6$

We know that middle term has the greatest binomial coefficient, Here $n = 6$

$\therefore$ Middle term $= \left(\frac{6}{2}+1\right)^{\text{th}}$ term

$= 4^{\text{th}}$ term $= T_4$

and given $T_4 = (n - 1) + T_3$

$T_{3+1} = (n - 1) + T_{2+1}$

${}^6C_3(3^{-x/4})^3 (3^{5x/4})^3 = (6 - 1) + {}^6C_2(3^{-x/4})^4 (3^{5x/4})^2$

$\Rightarrow \frac{6\cdot5\cdot4}{1\cdot2\cdot3} 3^{-3x/4}\cdot 3^{15x/4} = 5 + \frac{6\cdot5}{1\cdot2} 3^{-x}\cdot 3^{5x/2}$

$\Rightarrow \quad 20\cdot 3^{3x} = 5 + 15\cdot 3^{3x/2}$

Let $3^{3x/2} = t$ ....(i) $(t > 0)$

$\therefore \quad 20t^2 - 15t - 5 = 0$

$\Rightarrow \quad 4t^2 - 3t - 1 = 0 \Rightarrow (4t + 1)(t - 1) = 0$

$\therefore \quad t \neq -\frac{1}{4}$ $(\because \ t > 0)$

$\therefore \quad t = 1$ from (i), $3^{3x/2} = 1 = 3^0$

$\therefore \quad 3x/2 = 0 \Rightarrow x = 0$

2. **(0) :** Let $p(x) = ax^4 + bx^3 + cx^2 + dx + e$

$p'(x) = 4ax^3 + 3bx^2 + 2cx + d$

$p'(1) = 0, p'(2) = 0$

$\lim_{x\to0}\left(1+\frac{p(x)}{x^2}\right) = 2 \Rightarrow \lim_{x\to0}\left(\frac{x^2+p(x)}{x^2}\right) = 2$

So, $p(0) = 0 \Rightarrow e = 0$

Again $\lim_{x\to0}\left(\frac{2x+p'(x)}{2x}\right) = 2$

$\therefore \quad p'(0) = 0 \Rightarrow d = 0$

Again $\lim_{x\to0}\left(\frac{2x+p''(x)}{2}\right) = 2$

$p''(1) = 2 \Rightarrow 2c = 2 \ \therefore \ c = 1.$

$p(1) - p'(2) = 0$, gives on solution

$a = 1/4, b = -1$

$p(x) = \frac{x^4}{4} - x^3 + x^2$

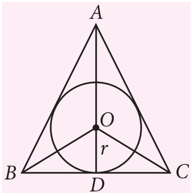
Hence $p(2) = 0$

3. **(1) :** $\frac{1}{81^n} - \frac{10}{81^n}\,{}^{2n}C_1 + \frac{10^2}{81^n}\,{}^{2n}C_2 - \frac{10^3}{81^n}\,{}^{2n}C_3 + \ldots\ldots + \frac{10^{2n}}{81^n}$

$= \frac{1}{81^n}[{}^{2n}C_0 + {}^{2n}C_1(-10) + {}^{2n}C_2(-10)^2 + \ldots + {}^{2n}C_n(-10)^{2n}]$

$= \frac{1}{81^n}\{1+(-10)\}^{2n} = \frac{(-1)^{2n}9^{2n}}{9^{2n}} = 1$

4. (5): Given, $A(z_1)=\dfrac{2i}{\sqrt{3}}$



$$B(z_2)=\frac{2}{\sqrt{3}}\left\{\frac{\sqrt{3}}{2}-i\frac{1}{2}\right\}=1-\frac{i}{\sqrt{3}}$$

and $$C(z_3)=\frac{2}{\sqrt{3}}\left\{-\frac{\sqrt{3}}{2}-\frac{i}{2}\right\}=-1-\frac{i}{\sqrt{3}}$$

Radius of incircle of ΔABC, *i.e.*, $r=\dfrac{1}{\sqrt{3}}$ units

Hence, any point on incircle *i.e.*, $P(z)$ is

$$\left(\frac{1}{\sqrt{3}}\cos\alpha,\frac{1}{\sqrt{3}}\sin\alpha\right)\ \ i.e.,\ \frac{1}{\sqrt{3}}(\cos\alpha+i\sin\alpha)$$

Solving for $|AP|^2+|BP|^2+|CP|^2$, $AP^2+BP^2+CP^2=5$

5. (1): $$\tan^{-1}\left(\frac{\sin\theta}{\sqrt{\cos 2\theta}}\right)=\sin^{-1}\left(\frac{\sin\theta}{\sqrt{\sin^2\theta+\cos 2\theta}}\right)$$

$$=\sin^{-1}\left(\frac{\sin\theta}{\sqrt{\cos^2\theta}}\right)=\sin^{-1}\left(\frac{\sin\theta}{\cos\theta}\right)\text{as } -\frac{\pi}{4}<\theta<\frac{\pi}{4}$$

Thus $$f(\theta)=\sin\left(\tan^{-1}\left(\frac{\sin\theta}{\sqrt{\cos^2\theta}}\right)\right)$$

$$=\sin\left(\sin^{-1}\left(\frac{\sin\theta}{\cos\theta}\right)\right)=\sin(\sin^{-1}(\tan\theta))=\tan\theta$$

Thus $d(f(\theta))=d(\tan\theta)$

6. (3): $$I=\int_2^8\frac{[x^2]dx}{[x^2-20x+100]+[x^2]}$$

$$I=\int_2^8\frac{[x^2]dx}{[(x-10)^2]+[x^2]}\qquad\text{....(i)}$$

Also, $$I=\int_2^8\frac{[(10-x^2)]dx}{[x^2]+[(10-x)^2]}\qquad\text{....(ii)}$$

$$\left(\because\ I=\int_a^b f(x)dx=\int_a^b f(a+b-x)dx\right)$$

(i) + (ii) gives $2I=\displaystyle\int_2^8 1dx=6$

$\Rightarrow\ I=3.$

7. (7): As $A=\{x|x^2+20\le 9x)\}$

$=\{x|x^2-9x+20\le 0)\}=\{x|(x-4)(x-5)\le 0)\}$

we have $A=[4,5]15$

$f(x)=2x^3-15x^2+36x-48$

$f'(x)=6x^2-30x+36=6(x-2)(x-3)$

f has no critical points in $[4, 5]$ as $f'(0)\neq 0$ in $(4, 5)$ and $f'(x)$ exists at all points.

$f(4)=2\cdot4^3-15\cdot4^2+36\cdot4-48=-16$

$f(5)=2\cdot5^3-15\cdot5^2+36\cdot5-48=7$

Thus the maximum value of f on $[4, 5]$ is 7.

8. (1): As $\sin^{-1}x\in\left(0,\dfrac{\pi}{2}\right)$

and $\cos^{-1}x=\dfrac{\pi}{2}-\sin^{-1}x\Rightarrow\ \cos^{-1}x\in\left(0,\dfrac{\pi}{2}\right)$

$$\Rightarrow\ \sin(\cos^{-1}x)=\cos(\sin^{-1}x)=\frac{1}{\sqrt{1-x^2}}$$

Thus, $\cos^{-1}(\sin(\cos^{-1}x))+\sin^{-1}(\cos(\sin^{-1}x))=\dfrac{\pi}{2}$

$\Rightarrow$ Required value $=\tan\dfrac{\pi}{4}=1$

9. (a, c) : Let the amount received by the sons be ₹ x, ₹ y and ₹ z respectively, then

$x\le y+z=101-x$

i.e. $2x\le 101$

$\therefore\ x\le 50,\ y\le 50,\ z\le 50$

$x+y+z=101$

The corresponding multinomial is $(1+x+...x^{50})^3$

$\therefore$ Coefficient of x^{101} in the expansion of

$(1+x+...x^{50})^3=1\times{}^{103}C_{101}-3\cdot{}^{52}C_{50}$

$={}^{103}C_2-3\cdot{}^{52}C_2$

10. (c) : $x(x^2+3y^2)dx+y(y^2+3x^2)dy=0$

$\Rightarrow\ x^3dx+y^3dy+3xy(ydx+xdy)=0$

$\Rightarrow\ x^3dx+y^3dy+3xy\,d(xy)=0$

$$\Rightarrow\ \frac{x^4}{4}+\frac{y^4}{4}+\frac{3(xy)^2}{2}+c'=0$$

11. (d): Since B is the inverse of A $\therefore$ $AB=I$

$$\begin{bmatrix}1&-1&1\\2&1&-3\\1&1&1\end{bmatrix}\times\frac{1}{10}\begin{bmatrix}4&2&2\\-5&0&\alpha\\1&-2&3\end{bmatrix}=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

$$i.e.,\ \begin{bmatrix}1&-1&1\\2&1&-3\\1&1&1\end{bmatrix}\begin{bmatrix}4&2&2\\-5&0&\alpha\\1&-2&3\end{bmatrix}=\begin{bmatrix}10&0&0\\0&10&0\\0&0&10\end{bmatrix}$$

Comparing $(1, 3)^{th}$ entry on both sides we get,

$\therefore\ 2-\alpha+3=0\ \Rightarrow\alpha=5.$

12. (b, d)

13. (a, b, c, d) : Since, the circle touches x-axis and its radius is 5, y-coordinate of the centre is 5 or -5. Circle also touches $3x-4y=0$.

Case I : When centre is $(h, 5)$, then

$\dfrac{(3h-20)}{5}=\pm 5$ or $3h-20=\pm 25$ or $h=15$ or $-5/3$

so centre is $(15, 5)$ or $(-5/3, 5)$

Case II : When centre is $(h, -5)$, then

$$\frac{(3h+20)}{5} = \pm 5 \text{ or } h = 5/3 \text{ or } -15$$

so centre is $(5/3, -5)$ or $(-15, -5)$

Hence, circles are

$$(x-15)^2 + (y-5)^2 = 25;\ \left(x+\frac{5}{3}\right)^2 + (y-5)^2 = 25;$$

$$\left(x-\frac{5}{3}\right)^2 + (y+5)^2 = 25;\ (x+15)^2 + (y+5)^2 = 25,$$

which are given in (a), (b), (c) and (d).

14. (a, b, c) : Taking log on both sides, we get

$y \log x = x \log y$ ...(i)

Differentiating, $\dfrac{dy}{dx}\log x + \dfrac{y}{x} = \log y + \dfrac{xdy}{ydx}$

$$\therefore\ \frac{dy}{dx} = \frac{\log y - y/x}{\log x - x/y} = \frac{y(x\log y - y)}{x(y\log x - x)} \quad \text{...(ii)}$$

$$= \frac{y^2(\log x - 1)}{x^2(\log y - 1)} \quad \text{[using (i)]} \quad \text{...(iii)}$$

Also, $\dfrac{yx^{y-1} - y^x \log y}{xy^{x-1} - x^y \log x} = \dfrac{(y/x)\cdot x^y - y^x \log y}{(x/y)\cdot y^x - x^y \log x}$

$$= \frac{\log y - y/x}{\log x - x/y} \quad [\because\ x^y = y^x]$$

$= dy/dx$ ... (iv)

(ii), (iii) and (iv) shows that (a), (b) and (c) all are correct.

15. (b, c) : Let $f(x) = ax^3 + bx^2 + cx + d$

$f(-x) = -ax^3 + bx^2 - cx + d$

Since $f(x)$ is odd

$\therefore\ ax^3 + bx^2 + cx + d \equiv ax^3 - bx^2 + cx - d$

$d = 0, b = 0$

$\therefore\ f(x) = ax^3 + cx$

$f'(x) = 3ax^2 + c = 0$ or $3ax^2 = -c$

$$\therefore\ x^2 = -\frac{c}{3a} > 0$$

a and c must be of different signs.

Non-zero roots of $f(x) = 0$ are $\pm\sqrt{\dfrac{-c}{a}}$

$\therefore\ \pm\sqrt{\dfrac{-c}{3a}}$ are closer to origin than the roots.

16. (a, b, c) : $f(x) = \begin{cases} x^2+2, & x<0 \\ 3, & x=0 \\ x+2, & x>0 \end{cases}$

$$f(0) = 3,\quad \lim_{x\to 0^-} f(x) = 2,\quad \lim_{x\to 0^+} f(x) = 2$$

$\therefore$ $f(x)$ has a maximum at $x = 0$

$f'(x) = 2x, x < 0$

$\therefore$ $f'(x) < 0$ for $x < 0$

$\therefore$ $f(x)$ is decreasing on the left of 0

$f''(x) = 2, x < 0$

$\therefore$ $f''(x) > 0, x < 0$

$\therefore$ $f'(x)$ is increasing on the left of 0.

17. (b, c, d) : $\cos^7 x + \sin^4 x = 1$

$\cos^7 x \le \cos^2 x$ and $\sin^4 x \le \sin^2 x$

$\therefore\ \cos^7 x + \sin^4 x \le \cos^2 x + \sin^2 x = 1$

The equality holds only if

$\cos^7 x = \cos^2 x$ and $\sin^4 x = \sin^2 x$

i.e. $\cos^2 x(1 - \cos^5 x) = 0$ and $\sin^2 x(1 - \sin^2 x) = 0$

i.e. $\cos x = 0$ or $\cos x = 1$ and

$\sin x = 0$ or $\sin x = \pm 1$

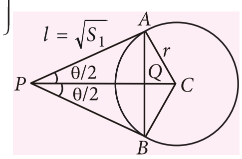
$\therefore\ \cos x = 0$ or $\cos x = 1$.

i.e. $x = \pm\pi/2$ and $x = 0$ $\therefore$ There are 3 solutions.

18. (b, c) : We have $\tan(\theta/2) = \dfrac{r}{\sqrt{S_1}} = \dfrac{\sqrt{g^2+f^2-c}}{\sqrt{S_1}}$

$$\text{or } \theta = 2\tan^{-1}\left\{\frac{\sqrt{g^2+f^2-c}}{\sqrt{S_1}}\right\}$$

$$= 2\cot^{-1}\left\{\frac{\sqrt{S_1}}{\sqrt{g^2+f^2-c}}\right\}.$$



19. (A) → (Q); (B) → (P); (C) → (R); (D) → (S)

20. (A) → (T); (B) → (P, R); (C) → (Q); (D) → (R)

PAPER-2

1. (1) : For $c < 1$; $\displaystyle\int_c^1 (8x^2 - x^5)dx = \frac{16}{3}$

$$\Rightarrow \frac{8}{3} - \frac{1}{6} - \frac{8c^3}{3} + \frac{c^6}{6} = \frac{16}{3}$$

$$\Rightarrow c^3\left[-\frac{8}{3} + \frac{c^3}{6}\right] = \frac{16}{3} - \frac{8}{3} + \frac{1}{6} = \frac{17}{6}$$

$\Rightarrow c = -1$

Again, for $c \ge 1$, none of the values of c satisfy the required condition that $\displaystyle\int_1^c (8x^2 - x^5)dx = \frac{16}{3}$

$\therefore\ c + 2 = 1$

2. (1) : $\sin^{-1}\left(\dfrac{1+x^2}{2x}\right) = \dfrac{\pi}{2}\sec(x-1)$

$\left|\frac{1+x^2}{2x}\right| \le 1$ *i.e.* $1+x^2 \le |2x|$

$\therefore$ $x = 1, -1$

Let $x = 1$ then $\sin^{-1}1 = \frac{\pi}{2}\sec 0 = \frac{\pi}{2}$

Let $x = -1$ then $\sin^{-1}(-1) = \frac{\pi}{2}\sec(-2)$

does not hold good. $\therefore$ only solution is $x = 1$.

3. (4) : $|8z_2z_3+27z_3z_1+64z_1z_2| = \left|z_1z_2z_3\left(\frac{8}{z_1}+\frac{27}{z_2}+\frac{64}{z_3}\right)\right|$

$= |z_1||z_2||z_3|\left|\frac{8\bar{z}_1}{|z_1|^2}+\frac{27\bar{z}_2}{|z_2|^2}+\frac{64\bar{z}_3}{|z_3|^2}\right|$

$= 2\cdot 3\cdot 4\cdot\left|\frac{8\bar{z}_1}{4}+\frac{27\bar{z}_2}{9}+\frac{64\bar{z}_3}{16}\right|$

$= 24\cdot|2\bar{z}_1+3\bar{z}_2+4\bar{z}_3|$ $\quad (\because |z| = |\bar{z}|)$

$= 24|\overline{2z_1+3z_2+4z_3}| = 24|2z_1+3z_2+4z_3| = 24\times 4$

$\Rightarrow$ $k = 4$

4. (7) : Since the normals are perpendicular

$\therefore$ the tangent will also be perpendicular to each other

$\therefore$ they will intersect on the directrix.

Equation of parabola is

$y^2 - 4y + 4 = 2x + 4$ or $(y-2)^2 = 2(x+2)$

$\therefore$ Equation of the directrix is

$x + 2 = -1/2$. *i.e.* $2x + 5 = 0$.

$\therefore$ $m + n = 7$

5. (2) : Number of ways of selecting one card out of 52 cards $n(S) = {}^{52}C_1 = 52$

$n(E)$ = number of selection of a card which is either a king or a queen $= {}^4C_1 + {}^4C_1 = 4 + 4 = 8$

Required probability $= 8/52 = 2/13$

So, $k = 2$

6. (5) **7. (4)** **8. (3)**

9. (a, c) : Let $\vec{r} = \lambda\vec{b} + \mu\vec{c}$

(a vector in the plane of $\vec{b}$ and $\vec{c}$)

$= (\lambda+\mu)\hat{i} + (2\lambda+\mu)\hat{j} - (\lambda+2\mu)\hat{k}$

Length of projection of $\vec{r}$ on $\vec{a} = \left|\frac{\vec{r}\cdot\vec{a}}{a}\right| = \sqrt{\frac{2}{3}}$

$\therefore$ $\frac{2(\lambda+\mu)-(2\lambda+\mu)-(\lambda+2\mu)}{\sqrt{6}} = \pm\frac{\sqrt{2}}{\sqrt{3}}$

$\therefore$ $-\lambda-\mu = \pm 2$ or, $\lambda+\mu = \pm 2$

If $\lambda + \mu = 2$, we have

$\vec{r} = 2\hat{i} + (\lambda+2)\hat{j} - (2+\mu)\hat{k}$

$\therefore$ For $\lambda = \mu = 1$, $\vec{r} = 2\hat{i}+3\hat{j}-3\hat{k}$ which is given in (a)

For (b), $\lambda + 2 = 3$ and $2 + \mu = -3$

$\lambda = 1, \mu = -5, \lambda + \mu \ne 2$

(b) is false.

For (d), $\lambda + 2 = 1$, $2 + \mu = 5$

$\Rightarrow$ $\lambda = -1, \mu = 3$

But $(\lambda + \mu) \ne 2$. $\therefore$ (d) is false.

If $\lambda + \mu = -2$, only possibility is (c),

so, $2\lambda + \mu = -1$ and $\lambda + 2\mu = -5$

$\Rightarrow$ $-2 + \lambda = -1$, $-2 + \mu = -5$

$\Rightarrow$ $\lambda = 1, \mu = -3$ $\Rightarrow$ $\lambda + \mu = -2$.

$\therefore$ (c) is correct.

10. (b, c, d) : By definition of signum function

$$\text{sgn}(x) = \begin{cases} 1, & \text{if } x > 0 \\ 0, & \text{if } x = 0 \\ -1, & \text{if } x < 0 \end{cases} \quad \therefore \text{(d) is correct}$$

$\therefore$ (d) is connected with $f(x)$ = sgn (x) also signum function can be defined by the choices (b) and (c) as

$$\frac{x}{|x|} \text{ or } \frac{|x|}{x} = \begin{cases} 1, & \text{if } x > 0 \\ -1, & \text{if } x < 0 \end{cases}$$

$$\therefore \text{ sgn}(x) = \begin{cases} \frac{|x|}{x} = \frac{x}{|x|} = 1, & \text{if } x > 0 \\ 0, & \text{if } x = 0 \\ \frac{|x|}{x} = \frac{x}{|x|} = -1, & \text{if } x < 0 \end{cases}$$

11. (b, d) : Clearly $x^2 + 4x + \alpha^2 - \alpha \geq 0\ \forall\ x \in R$ and must take all values of the interval $[0, \infty)$

$\Rightarrow \quad D = 0$

i.e. $16 - 4(\alpha^2 - \alpha) = 0 \ \Rightarrow \ \alpha^2 - \alpha = 4$

$\Rightarrow \quad \alpha = \frac{1 \pm \sqrt{17}}{2}$.

12. (a, b) : $f(x) = \sqrt{2} \sin\left(\frac{\{x\}}{a} + \frac{\pi}{4}\right)$

$\Rightarrow \quad \{x\} = \left(2n\pi + \frac{\pi}{4}\right)a \ \therefore \ 0 < \left(2n\pi + \frac{\pi}{4}\right)a < 1$

$\therefore \quad 0 < a < \frac{4}{(8n+1)\pi} \quad \therefore \quad 0 < a < \frac{4}{\pi}$.

13. (a) : ${}^{2n}C_0 + {}^{2n}C_1 + ... + {}^{2n}C_n$

$= {}^{2n}C_0 + {}^{2n}C_1 + ... + {}^{2n}C_{2n} - ({}^{2n}C_{n+1} + ... + {}^{2n}C_{2n})$

$= 2^{2n} - ({}^{2n}C_{n-1} + {}^{2n}C_{n-2} + ... + {}^{2n}C_0)$

$= 2^{2n} - ({}^{2n}C_0 + {}^{2n}C_1 + ... + {}^{2n}C_{n-1} + {}^{2n}C_n - {}^{2n}C_n)$

$\therefore \quad 2({}^{2n}C_0 + {}^{2n}C_1 + ... {}^{2n}C_n) = 2^{2n} + {}^{2n}C_n$

$\therefore \quad {}^{2n}C_0 + {}^{2n}C_1 + ... + {}^{2n}C_n$

$= 2^{2n-1} + \frac{2n\{(2n-1)!\}}{2n(n!)\{(n-1)!\}} = 2^{2n-1} + \frac{(2n-1)!}{n!\,(n-1)!}$.

14. (a, d) **15. (b, c)** **16. (c,d)**

17. (b): $\int x \cos x\, dx$

Let $I = \int x \cos x\, dx$

Now taking x as first function and $\cos x$ as second function, apply method of integration by parts.

$I = x\left(\int \cos x\ dx\right) - \int\left[\left\{\frac{d}{dx}(x)\right\}\int \cos x dx\right]dx$

$\Rightarrow I = x \sin x - \int 1 \cdot \sin x\, dx \ \therefore \ I = x \sin x + \cos x + C$

18. (c) : $I = \int \underset{\text{I}}{\log_e |x|}\, dx = \int \log_e |x| \,.\, \underset{\text{II}}{1}\, dx$

$\Rightarrow \quad I = \log|x| \cdot x - \int \frac{1}{x} \cdot x dx = x \log|x| - \int 1 \cdot dx$

$\Rightarrow \quad I = x \log|x| - x + C$

19. (b): As y_1, y_2 are the solutions of the differential equation

$\therefore \quad \frac{dy}{dx} + p(x) \cdot y = Q(x)$ ...(i)

$\frac{dy_1}{dx} + p(x) \cdot y_1 = Q(x)$ ...(ii)

$\frac{dy_2}{dx} + p(x) \cdot y_2 = Q(x)$ ...(iii)

From (i) and (ii)

$\left(\frac{dy}{dx} - \frac{dy_1}{dx}\right) + p(x) \cdot (y - y_1) = 0$

$\frac{d}{dx}(y - y_1) + p(x) \cdot (y - y_1) = 0$ ...(iv)

From (ii) and (iii)

$\frac{d}{dx}(y_1 - y_2) + p(x) \cdot (y_1 - y_2) = 0$ ...(v)

From (iv) and (v)

$$\frac{\frac{d}{dx}(y - y_1)}{\frac{d}{dx}(y_1 - y_2)} = \frac{y - y_1}{y_1 - y_2}$$

$$\Rightarrow \frac{\frac{d}{dx}(y - y_1)}{y - y_1} = \frac{\frac{d}{dx}(y_1 - y_2)}{y_1 - y_2}$$

On integrating we get

$\log(y - y_1) = \log(y_1 - y_2) + \log c \Rightarrow y = y_1 + c(y_1 - y_2)$

20. (a) : Given $y = \alpha y_1 + \beta y_2$ is also a solution

$\therefore \quad \frac{d}{dx}(\alpha y_1 + \beta y_2) + \beta\left(\frac{dy_2}{dx} + p(x) \cdot y_2\right) = Q(x)$

$\Rightarrow \alpha\left(\frac{dy_1}{dx} + p(x) y_1\right) + \beta\left(\frac{dy_2}{dx} + p(x) \cdot y_2\right) = Q(x)$

$\Rightarrow \quad \alpha Q(x) + \beta\, Q(x) = Q(x) \ \Rightarrow \ \alpha + \beta = 1.$

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CBSE BOARD SOLVED PAPER 2016

CLASS XII

Units	VSA(1 mark)	SA(4 marks)	VBQ(4 marks)	LA(6 marks)	Total
Relations and Functions	---	---	---	6(1)	6(1)
Inverse Trigonometric Functions	---	4(1)	---	---	4(1)
Matrices	2(2)	---	---	---	2(2)
Determinants	1(1)	---	4(1)	6(1)	11(3)
Continuity and Differentiability	---	8(2)	---	---	8(2)
Application of Derivatives	---	4(1)	---	6(1)	10(2)
Integrals	---	12(3)	---	---	12(3)
Application of Integrals	---	---	---	6(1)	6(1)
Differential Equations	---	8(2)	---	---	8(2)
Vector Algebra	2(2)	4(1)	---	---	6(3)
Three Dimensional Geometry	1(1)	4(1)	---	6(1)	11(3)
Linear Programming	---	---	---	6(1)	6(1)
Probability	---	4(1)	---	6(1)	10(2)
Total	**6(6)**	**48(12)**	**4(1)**	**42(7)**	**100(26)**

Time Allowed : 3 hours **Maximum Marks : 100**

GENERAL INSTRUCTIONS

(i) All questions are compulsory.
(ii) Please check that this question paper contains **26** questions.
(iii) Questions **1-6** in Section A are very short-answer type questions carrying **1** mark each.
(iv) Questions **7-19** in Section B are long-answer **I** type questions carrying **4** marks each.
(v) Questions **20-26** in Section C are long-answer **II** type questions carrying **6** marks each.
(vi) Please write down the serial number of the question before attempting it.

SECTION - A

1. The two vectors $\hat{j}+\hat{k}$ and $3\hat{i}-\hat{j}+4\hat{k}$ represent the two sides AB and AC, respectively of a ΔABC. Find the length of the median through A.

2. Find the vector equation of a plane which is at a distance of 5 units from the origin and its normal vector is $2\hat{i}-3\hat{j}+6\hat{k}$.

3. Find the maximum value of $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+\sin\theta & 1 \\ 1 & 1 & 1+\cos\theta \end{vmatrix}$

4. If A is a square matrix such that $A^2 = I$, then find the simplified value of $(A - I)^3 + (A + I)^3 - 7A$.

5. Matrix $A=\begin{bmatrix}0 & 2b & -2\\ 3 & 1 & 3\\ 3a & 3 & -1\end{bmatrix}$ is given to be symmetric, find values of a and b.

6. Find the position vector of a point which divides the join of points with position vectors $\vec{a}-2\vec{b}$ and $2\vec{a}+\vec{b}$ externally in the ratio 2 : 1.

SECTION - B

7. Find the general solution of the following differential equation : $(1+y^2)+(x-e^{\tan^{-1}y})\frac{dy}{dx}=0$

8. Show that the vectors $\vec{a}, \vec{b}$ and $\vec{c}$ are coplanar if $\vec{a}+\vec{b}, \vec{b}+\vec{c}$ and $\vec{c}+\vec{a}$ are coplanar.

9. Find the vector and cartesian equations of the line through the point (1, 2, –4) and perpendicular to the two lines
$\vec{r}=(8\hat{i}-19\hat{j}+10\hat{k})+\lambda(3\hat{i}-16\hat{j}+7\hat{k})$ and
$\vec{r}=(15\hat{i}+29\hat{j}+5\hat{k})+\mu(3\hat{i}+8\hat{j}-5\hat{k})$.

10. Three persons A, B and C apply for a job of Manager in a Private Company. Chances of their selection (A, B and C) are in the ratio 1 : 2 : 4. The probabilities that A, B and C can introduce changes to improve profits of the company are 0.8, 0.5 and 0.3 respectively. If the change does not take place, find the probability that it is due to the appointment of C.

OR

A and B throw a pair of dice alternately. A wins the game if he gets a total of 7 and B wins the game if he gets a total of 10. If A starts the game, then find the probability that B wins.

11. Prove that : $\tan^{-1}\frac{1}{5}+\tan^{-1}\frac{1}{7}+\tan^{-1}\frac{1}{3}+\tan^{-1}\frac{1}{8}=\frac{\pi}{4}$

OR

Solve for x : $2\tan^{-1}(\cos x)=\tan^{-1}(2\operatorname{cosec} x)$

12. The monthly incomes of Aryan and Babban are in the ratio 3 : 4 and their monthly expenditures are in the ratio 5 : 7. If each saves ₹15,000 per month, find their monthly incomes using matrix method. This problem reflects which value?

13. If $x=a\sin 2t(1+\cos 2t)$ and $y=b\cos 2t(1-\cos 2t)$, find the values of $\frac{dy}{dx}$ at $t=\frac{\pi}{4}$ and $t=\frac{\pi}{3}$.

OR

If $y=x^x$, prove that $\frac{d^2y}{dx^2}-\frac{1}{y}\left(\frac{dy}{dx}\right)^2-\frac{y}{x}=0$.

14. Find the values of p and q, for which

$$f(x)=\begin{cases}\dfrac{1-\sin^3 x}{3\cos^2 x}, & \text{if } x<\pi/2\\ p, & \text{if } x=\pi/2\\ \dfrac{q(1-\sin x)}{(\pi-2x)^2}, & \text{if } x>\pi/2\end{cases}$$

is continuous at $x=\pi/2$.

15. Show that the equation of normal at any point t on the curve $x = 3\cos t - \cos^3 t$ and $y = 3\sin t - \sin^3 t$ is $4(y\cos^3 t - x\sin^3 t) = 3\sin 4t$.

16. Find $\int \frac{(3\sin\theta-2)\cos\theta}{5-\cos^2\theta-4\sin\theta}d\theta$.

OR

Evaluate $\int_0^{\pi} e^{2x}\cdot\sin\left(\frac{\pi}{4}+x\right)dx$.

17. Find $\int \frac{\sqrt{x}}{\sqrt{a^3-x^3}}dx$.

18. Evaluate $\int_{-1}^{2}|x^3-x|dx$.

19. Find the particular solution of the differential equation $(1 - y^2)(1 + \log x)dx + 2xy\,dy = 0$, given that $y = 0$ when $x = 1$.

SECTION - C

20. Find the coordinate of the point P where the line through $A(3, -4, -5)$ and $B(2, -3, 1)$ crosses the plane passing through three points $L(2, 2, 1)$, $M(3, 0, 1)$ and $N(4, -1, 0)$. Also, find the ratio in which P divides the line segment AB.

21. An urn contains 3 white and 6 red balls. Four balls are drawn one by one with replacement from the urn. Find the probability distribution of the number of red balls drawn. Also find mean and variance of the distribution.

22. A manufacturer produces two products A and B. Both the products are processed on two different machines. The available capacity of first machine is 12 hours and that of second machine is 9 hours

per day. Each unit of product A requires 3 hours on both machines and each unit of product B requires 2 hours on first machine and 1 hour on second machine. Each unit of product A is sold at ₹7 profit and that of B at a profit of ₹4. Find the production level per day for maximum profit graphically.

23. Let $f : N \to N$ be a function defined as $f(x) = 9x^2 + 6x - 5$. Show that $f : N \to S$, where S is the range of f, is invertible. Find the inverse of f and hence find $f^{-1}(43)$ and $f^{-1}(163)$.

24. Prove that $\begin{vmatrix} yz-x^2 & zx-y^2 & xy-z^2 \\ zx-y^2 & xy-z^2 & yz-x^2 \\ xy-z^2 & yz-x^2 & zx-y^2 \end{vmatrix}$ is divisible by $(x + y + z)$, and hence find the quotient.

OR

Using elementary transformations, find the inverse of the matrix $A = \begin{bmatrix} 8 & 4 & 3 \\ 2 & 1 & 1 \\ 1 & 2 & 2 \end{bmatrix}$ and use it to solve the following system of linear equations:

$8x + 4y + 3z = 19$

$2x + y + z = 5$

$x + 2y + 2z = 7$

25. Show that the altitude of the right circular cone of maximum volume that can be inscribed in a sphere of radius r is $\frac{4r}{3}$. Also find maximum volume in terms of volume of the sphere.

OR

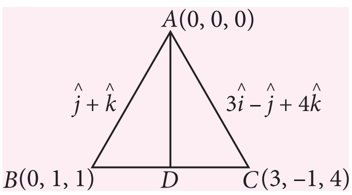
Find the intervals in which $f(x) = \sin 3x - \cos 3x$, $0 < x < \pi$, is strictly increasing or strictly decreasing.

26. Using integration find the area of the region $\{(x, y) : x^2 + y^2 \le 2ax, y^2 \ge ax, x, y \ge 0\}$

SOLUTIONS

1. Take A to be as origin (0, 0, 0).

$\therefore$ Coordinates of B are (0, 1, 1) and coordinates of C are (3, –1, 4).



$\because$ D is the mid point of BC.

$\therefore$ Coordinates of D are $\left(\frac{3}{2}, 0, \frac{5}{2}\right)$

So, length of $AD = \sqrt{\left(0-\frac{3}{2}\right)^2 + (0)^2 + \left(0-\frac{5}{2}\right)^2}$

$= \sqrt{\frac{9}{4} + \frac{25}{4}} = \frac{\sqrt{34}}{2}$ units

2. We know that perpendicular distance of a point with position vector $\vec{a}$ from the plane $\vec{r} \cdot \vec{n} = d$ is $\left|\frac{\vec{a} \cdot \vec{n} - d}{|\vec{n}|}\right|$

Here, $\vec{a} = 0\hat{i} + 0\hat{j} + 0\hat{k}$, $\vec{n} = 2\hat{i} - 3\hat{j} + 6\hat{k}$

According to question,

$$\left|\frac{0-d}{\sqrt{4+9+36}}\right| = 5$$

$\Rightarrow$ $d = 35$

$\therefore$ Required equation of plane is

$\vec{r} \cdot (2\hat{i} - 3\hat{j} + 6\hat{k}) = 35$

3. Let $\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+\sin\theta & 1 \\ 1 & 1 & 1+\cos\theta \end{vmatrix}$

Applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$, we get

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 0 & \sin\theta & 0 \\ 0 & 0 & \cos\theta \end{vmatrix} = \sin\theta\cos\theta$$

$\therefore$ Maximum value of Δ is $\frac{1}{2}$.

4. Given, $A^2 = I$

Consider, $(A - I)^3 + (A + I)^3 - 7A$

$= A^3 - I^3 - 3A^2I + 3AI^2 + A^3 + I^3 + 3A^2I + 3AI^2 - 7A$

$= 2A^3 + 6AI^2 - 7A = 2AA^2 + 6AI - 7A$

$= 2AI + 6A - 7A = 2A + 6A - 7A = A$

5. Given, $A = \begin{bmatrix} 0 & 2b & -2 \\ 3 & 1 & 3 \\ 3a & 3 & -1 \end{bmatrix}$

$\because$ A is symmetric.

$\therefore$ $A' = A$

$$\Rightarrow \begin{bmatrix} 0 & 3 & 3a \\ 2b & 1 & 3 \\ -2 & 3 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 2b & -2 \\ 3 & 1 & 3 \\ 3a & 3 & -1 \end{bmatrix}$$

On comparing, we get

$a = \dfrac{-2}{3}$ and $b = \dfrac{3}{2}$

6. Required position vector $= \dfrac{2\cdot(2\vec{a}+\vec{b})-1(\vec{a}-2\vec{b})}{2-1}$

$$= \frac{4\vec{a}+2\vec{b}-\vec{a}+2\vec{b}}{1} = 3\vec{a}+4\vec{b}$$

7. We have, $(1+y^2)+(x-e^{\tan^{-1}y})\dfrac{dy}{dx} = 0$

$$\Rightarrow \quad (x-e^{\tan^{-1}y})\frac{dy}{dx} = -(1+y^2)$$

$$\Rightarrow \quad \frac{dx}{dy} = \frac{x-e^{\tan^{-1}y}}{-(1+y^2)}$$

$$\Rightarrow \quad \frac{dx}{dy}+\frac{1}{1+y^2}\cdot x = \frac{e^{\tan^{-1}y}}{1+y^2}$$

This is a homogeneous linear differential equation.

$$\text{I.F.} = e^{\int \frac{dy}{1+y^2}} = e^{\tan^{-1}y}$$

$\therefore$ Solution is

$$xe^{\tan^{-1}y} = \int \frac{(e^{\tan^{-1}y})^2}{1+y^2}\,dy + c$$

$$= \int \frac{e^{2\tan^{-1}y}}{1+y^2}\,dy + c$$

$$\Rightarrow \quad x\cdot e^{\tan^{-1}y} = \frac{e^{2\tan^{-1}y}}{2} + c_1$$

$$\Rightarrow \quad x = \frac{e^{\tan^{-1}y}}{2} + c_1 e^{-\tan^{-1}y}$$

8. Since, $\vec{a}+\vec{b}$, $\vec{b}+\vec{c}$ and $\vec{c}+\vec{a}$ are coplanar.

$$\therefore \quad (\vec{a}+\vec{b})\cdot\left[(\vec{b}+\vec{c})\times(\vec{c}+\vec{a})\right] = 0$$

$$\Rightarrow \quad (\vec{a}+\vec{b})\cdot\left[\vec{b}\times\vec{c}+\vec{b}\times\vec{a}+\vec{c}\times\vec{c}+\vec{c}\times\vec{a}\right] = 0$$

$$\Rightarrow \quad (\vec{a}+\vec{b})\cdot(\vec{b}\times\vec{c}+\vec{b}\times\vec{a}+\vec{c}\times\vec{a}) = 0 \quad \left[\because\ \vec{c}\times\vec{c} = 0\right]$$

$$\Rightarrow \quad \vec{a}\cdot(\vec{b}\times\vec{c})+\vec{a}\cdot(\vec{b}\times\vec{a})+\vec{a}\cdot(\vec{c}\times\vec{a})+\vec{b}\cdot(\vec{b}\times\vec{c}) + \vec{b}\cdot(\vec{b}\times\vec{a})+\vec{b}\cdot(\vec{c}\times\vec{a}) = 0$$

$$\Rightarrow \quad 2\left[\vec{a}\cdot(\vec{b}\times\vec{c})\right] = 0 \Rightarrow \vec{a}\cdot(\vec{b}\times\vec{c}) = 0$$

$\Rightarrow$ $\vec{a}$, $\vec{b}$ and $\vec{c}$ are coplanar.

9. The required line is perpendicular to the lines which are parallel to vectors $\vec{b}_1 = 3\hat{i}-16\hat{j}+7\hat{k}$ and $\vec{b}_2 = 3\hat{i}+8\hat{j}-5\hat{k}$

So, it is parallel to the vector $\vec{b} = \vec{b}_1\times\vec{b}_2$

Now, $\vec{b} = \vec{b}_1\times\vec{b}_2 = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3 & -16 & 7\\ 3 & 8 & -5\end{vmatrix} = 24\hat{i}+36\hat{j}+72\hat{k}$

So, vector equation of the line passing through the point (1, 2, – 4) and parallel to the vector $\vec{b} = 24\hat{i}+36\hat{j}+72\hat{k}$ is

$$\vec{r} = (\hat{i}+2\hat{j}-4\hat{k})+s(24\hat{i}+36\hat{j}+72\hat{k})$$

Cartesian equation is,

$$x\hat{i}+y\hat{j}+z\hat{k} = \hat{i}+2\hat{j}-4\hat{k}+s(24\hat{i}+36\hat{j}+72\hat{k})$$

$$\Rightarrow \quad \frac{x-1}{24} = \frac{y-2}{36} = \frac{z+4}{72} \Rightarrow \frac{x-1}{2} = \frac{y-2}{3} = \frac{z+4}{6}$$

10. Let I = Event that improvement changes take place.

Probability of selection of A, $P(A) = \dfrac{1}{7}$

Probability of selection of B, $P(B) = \dfrac{2}{7}$

Probability of selection of C, $P(C) = \dfrac{4}{7}$

Probability that A introduce changes to improve, $P(I/A) = 0.8$

Probability that B introduce changes to improve, $P(I/B) = 0.5$

Probability that C introduce changes to improve, $P(I/C) = 0.3$

Probability that A does not introduce changes, $P(\bar{I}/A) = 1-0.8 = 0.2$

Probability that B does not introduce changes, $P(\bar{I}/B) = 1-0.5 = 0.5$

Probability that C does not introduce changes $P(\bar{I}/C) = 1-0.3 = 0.7$

So, required probability,

$$P(C/\bar{I}) = \frac{P(C)P(\bar{I}/C)}{P(A)P(\bar{I}/A)+P(B)P(\bar{I}/B)+P(C)P(\bar{I}/C)}$$

$$= \frac{\frac{4}{7}\times 0.7}{\frac{1}{7}\times 0.2+\frac{2}{7}\times 0.5+\frac{4}{7}\times 0.7} = 0.7$$

OR

Total outcomes = 36

Favourable outcomes for A to win are

{(1, 6), (6, 1), (2, 5), (5, 2), (3, 4), (4, 3)}

$\therefore$ Probability of A to win, $P(A) = \dfrac{6}{36} = \dfrac{1}{6}$

Probability of A to lose, $P(\bar{A}) = 1 - \frac{1}{6} = \frac{5}{6}$

Fovourable outcomes for B to win are

$\{(4, 6), (6, 4), (5, 5)\}$

$\therefore$ Probability of B to win, $P(B) = \frac{3}{36} = \frac{1}{12}$

Probability of B to lose, $P(\bar{B}) = 1 - \frac{1}{12} = \frac{11}{12}$

Required probability

$$= P(\bar{A})P(B) + P(\bar{A})P(\bar{B})P(\bar{A})P(B) + P(\bar{A})P(\bar{B})P(\bar{A})P(\bar{B})P(\bar{A})P(B) + \ldots$$

$$= \frac{5}{6} \times \frac{1}{12} + \frac{5}{6} \times \frac{11}{12} \times \frac{5}{6} \times \frac{1}{12} + \frac{5}{6} \times \frac{11}{12} \times \frac{5}{6} \times \frac{11}{12} \times \frac{5}{6} \times \frac{1}{12} + \ldots$$

$$= \frac{5/72}{1 - \frac{5}{6} \times \frac{11}{12}} = \frac{5}{17}$$

11. L.H.S. $= \tan^{-1}\frac{1}{5} + \tan^{-1}\frac{1}{7} + \tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{8}$

$$= \tan^{-1}\left(\frac{\frac{1}{5}+\frac{1}{7}}{1-\frac{1}{5}\cdot\frac{1}{7}}\right) + \tan^{-1}\left(\frac{\frac{1}{3}+\frac{1}{8}}{1-\frac{1}{3}\cdot\frac{1}{8}}\right)$$

$$= \tan^{-1}\left(\frac{12}{34}\right) + \tan^{-1}\left(\frac{11}{23}\right)$$

$$= \tan^{-1}\left(\frac{\frac{6}{17}+\frac{11}{23}}{1-\frac{6}{17}\cdot\frac{11}{23}}\right) = \tan^{-1}\left(\frac{325}{325}\right) = \tan^{-1}(1) = \frac{\pi}{4}$$

OR

Given equation is

$2\tan^{-1}(\cos x) = \tan^{-1}(2\,\text{cosec}\, x)$ ...(i)

Let $2\tan^{-1}(\cos x) = \theta \Rightarrow \cos x = \tan(\theta/2)$ ...(ii)

Now,

$$2\,\text{cosec}\, x = \frac{2}{\sin x} = \frac{2}{\sqrt{1-\cos^2 x}} = \frac{2}{\sqrt{1-\tan^2(\theta/2)}} \quad \text{...(iii)}$$

From (ii) and (iii), given equation becomes

$$\theta = \tan^{-1}\left[\frac{2}{\sqrt{1-\tan^2\frac{\theta}{2}}}\right] \Rightarrow \tan\theta = \frac{2}{\sqrt{1-\tan^2\frac{\theta}{2}}}$$

$$\Rightarrow \frac{2\tan(\theta/2)}{1-\tan^2(\theta/2)} = \frac{2}{\sqrt{1-\tan^2(\theta/2)}}$$

$$\Rightarrow \tan(\theta/2) = \sqrt{1-\tan^2(\theta/2)} \quad \text{...(iv)}$$

$$\Rightarrow \tan^2(\theta/2) = 1 - \tan^2(\theta/2) \Rightarrow 2\tan^2(\theta/2) = 1$$

$$\Rightarrow \tan\frac{\theta}{2} = \frac{1}{\sqrt{2}} \quad \left[\because \text{From(iv)}, \tan\frac{\theta}{2} > 0\right]$$

$$\Rightarrow \cos x = \frac{1}{\sqrt{2}} \quad \text{[From (ii)]}$$

$$\therefore \quad x = 2n\pi \pm \frac{\pi}{4}, n \in Z = \pm\frac{\pi}{4}, \frac{7\pi}{4}, \frac{9\pi}{4}, \ldots$$

But for equation (i) to be satisfied cosec x and cos x must have same sign.

$\therefore$ x lies in 1st quadrant.

$\Rightarrow$ $x = \pi/4$

12. Let the monthly income of Aryan be ₹ $3x$ and that of Babban be ₹ $4x$

Also, let monthly expenditure of Aryan be ₹ $5y$ and that of Babban be ₹ $7y$

According to question,

$3x - 5y = 15000$

$4x - 7y = 15000$

These equations can be rewritten as

$$\begin{bmatrix} 3 & -5 \\ 4 & -7 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 15000 \\ 15000 \end{bmatrix}$$

Applying $R_2 \to R_2 + \left(-\frac{4}{3}\right)R_1$, we get

$$\begin{bmatrix} 3 & -5 \\ 0 & -1/3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 15000 \\ -5000 \end{bmatrix}$$

So, $\frac{-1}{3}y = -5000 \Rightarrow y = 15000$

$3x - 5y = 15000$

$\Rightarrow$ $3x = 15000 + 75000 \Rightarrow x = 30000$

So, monthly income of Aryan = 3 × 30000 = ₹90000

Monthly income of Babban = 4 × 30000 = ₹120000

13. $x = a\sin 2t\,(1 + \cos 2t)$

$y = b\cos 2t\,(1 - \cos 2t)$

Now, $\frac{dx}{dt} = 2a\cos 2t(1+\cos 2t) + a\sin 2t(-2\sin 2t)$

$= 2a\cos 2t + 2a[\cos^2 2t - \sin^2 2t]$

$= 2a\cos 2t + 2a\cos 4t$

and $\frac{dy}{dt} = -2b\sin 2t(1-\cos 2t) + b\cos 2t(2\sin 2t)$

$= -2b\sin 2t + 4b(\sin 2t\cos 2t)$

$= -2b\sin 2t + 2b\sin 4t$

So, $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{2b(\sin 4t - \sin 2t)}{2a(\cos 4t + \cos 2t)}$

$$\left[\frac{dy}{dx}\right]_{\text{at } t=\pi/4} = \frac{b}{a}\left[\frac{\sin\pi - \sin(\pi/2)}{\cos\pi + \cos(\pi/2)}\right]$$

$$= \frac{b}{a}\left[\frac{0-1}{-1+0}\right] = \frac{b}{a}$$

$$\left[\frac{dy}{dx}\right]_{\text{at } t=\pi/3} = \frac{b}{a}\left[\frac{\sin(4\pi/3) - \sin(2\pi/3)}{\cos(4\pi/3) + \cos(2\pi/3)}\right]$$

$$= \frac{b}{a}\left[\frac{\frac{-\sqrt{3}}{2} - \frac{\sqrt{3}}{2}}{\frac{-1}{2} - \frac{1}{2}}\right] = \frac{b}{a}\left[\frac{-\sqrt{3}}{-1}\right] = \frac{\sqrt{3}b}{a}$$

OR

We have, $y = x^x$

$\Rightarrow \quad y = e^{x\log x}$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = e^{x\log x}\left(x \times \frac{1}{x} + \log x\right)$$

$$\Rightarrow \quad \frac{dy}{dx} = x^x(1 + \log x)$$

$$\Rightarrow \quad \frac{dy}{dx} = y(1+\log x) \Rightarrow (1 + \log x) = \frac{1}{y}\cdot\left(\frac{dy}{dx}\right) \quad \text{...(i)}$$

Again differentiating w.r.t. x, we get

$$\frac{d^2y}{dx^2} = (1+\log x)\cdot\frac{dy}{dx} + y \times \frac{1}{x}$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} = \frac{1}{y}\left(\frac{dy}{dx}\right)^2 + \frac{y}{x} \qquad \text{[From (i)]}$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} - \frac{1}{y}\left(\frac{dy}{dx}\right)^2 - \frac{y}{x} = 0. \text{ Hence proved}$$

14. $\because$ $f(x)$ is continuous at $\pi/2$.

$$\Rightarrow \quad \lim_{x\to\pi/2^-} f(x) = \lim_{x\to\pi/2^+} f(x) = f(\pi/2) \quad \text{...(i)}$$

Now, $\displaystyle\lim_{x\to\pi/2^-} f(x) = \lim_{h\to 0} f\left(\frac{\pi}{2} - h\right)$

$$= \lim_{h\to 0}\frac{1-\sin^3\left(\frac{\pi}{2}-h\right)}{3\cos^2\left(\frac{\pi}{2}-h\right)} = \lim_{h\to 0}\frac{1-\cos^3 h}{3\sin^2 h}$$

$$= \lim_{h\to 0}\frac{(1-\cos h)(1+\cos^2 h+\cos h)}{3(1-\cos h)(1+\cos h)}$$

$$= \lim_{h\to 0}\frac{(1+\cos^2 h+\cos h)}{3(1+\cos h)} = \frac{1+1+1}{3(1+1)} = \frac{1}{2}$$

Now, $\displaystyle\lim_{x\to\pi/2^+} f(x) = \lim_{h\to 0} f\left(\frac{\pi}{2} + h\right)$

$$= \lim_{h\to 0}\frac{q\left[1-\sin\left(\frac{\pi}{2}+h\right)\right]}{\left[\pi - 2\left(\frac{\pi}{2}+h\right)\right]^2} = \lim_{h\to 0}\frac{q(1-\cos h)}{4h^2}$$

$$= \frac{q}{4} \times \lim_{h\to 0}\frac{2\sin^2\frac{h}{2}}{\frac{h^2}{4}\times 4} = \frac{q}{4} \times \frac{2}{4} = \frac{q}{8} \text{ and } f(\pi/2) = p$$

Hence, from (i)

$$\frac{1}{2} = p = \frac{q}{8} \Rightarrow p = \frac{1}{2} \text{ and } q = 4$$

15. $x = 3\cos t - \cos^3 t$ and $y = 3\sin t - \sin^3 t$

Now, $\dfrac{dx}{dt} = -3\sin t + 3\cos^2 t\sin t = -3\sin t(1-\cos^2 t)$

$= -3\sin t\sin^2 t = -3\sin^3 t$

Also, $\dfrac{dy}{dt} = 3\cos t - 3\sin^2 t\cos t = 3\cos t(1-\sin^2 t)$

$= 3\cos t\cos^2 t = 3\cos^3 t$

So, $\dfrac{dy}{dx} = \dfrac{dy/dt}{dx/dt} = \dfrac{3\cos^3 t}{-3\sin^3 t} = -\dfrac{\cos^3 t}{\sin^3 t}$

Slope of normal $= \dfrac{-1}{\frac{dy}{dx}} = \dfrac{\sin^3 t}{\cos^3 t}$

Required equation of normal is

$$y - (3\sin t - \sin^3 t) = \frac{\sin^3 t}{\cos^3 t}\left[x - (3\cos t - \cos^3 t)\right]$$

$\Rightarrow \quad y\cos^3 t - 3\sin t\cos^3 t + \sin^3 t\cos^3 t$

$= x\sin^3 t - 3\cos t\sin^3 t + \sin^3 t\cos^3 t$

$\Rightarrow \quad y\cos^3 t - x\sin^3 t = 3\sin t\cos t(\cos^2 t - \sin^2 t)$

$$\Rightarrow \quad y\cos^3 t - x\sin^3 t = \frac{3\sin 2t\cdot\cos 2t}{2}$$

$$\Rightarrow \quad y\cos^3 t - x\sin^3 t = \frac{3}{2}\cdot\frac{\sin 4t}{2}$$

$\Rightarrow \quad 4(y\cos^3 t - x\sin^3 t) = 3\sin 4t$. Hence proved

16. Let $I = \displaystyle\int\frac{(3\sin\theta - 2)\cos\theta}{5-\cos^2\theta - 4\sin\theta}\,d\theta$

$$= 3\int\frac{\sin\theta\cos\theta}{4+\sin^2\theta - 4\sin\theta}\,d\theta - 2\int\frac{\cos\theta}{4+\sin^2\theta-4\sin\theta}\,d\theta$$

$= 3I_1 - 2I_2$ (say)

Now, $I_1 = \displaystyle\int\frac{\sin\theta\cos\theta}{4+\sin^2\theta-4\sin\theta}\,d\theta$

Put $\sin^2\theta = t \Rightarrow 2\sin\theta\cos\theta\, d\theta = dt$

$$\therefore\quad I_1 = \frac{1}{2}\int \frac{dt}{4+t-4\sqrt{t}} = \frac{1}{2}\int \frac{dt}{(\sqrt{t}-2)^2}$$

Put $\sqrt{t}-2 = u \Rightarrow \sqrt{t} = u+2$

$$\Rightarrow\quad \frac{1}{2\sqrt{t}}dt = du \Rightarrow dt = 2(u+2)du$$

$$\therefore\quad I_1 = \int \frac{(u+2)}{u^2}du = \int \frac{du}{u} + 2\int \frac{du}{u^2}$$

$$= \log u - \frac{2}{u} + c_1 = \log(\sqrt{t}-2) - \frac{2}{\sqrt{t}-2} + c_1$$

$$= \log(\sin\theta - 2) - \frac{2}{\sin\theta - 2} + c_1$$

$$I_2 = \int \frac{\cos\theta}{4+\sin^2\theta - 4\sin\theta}\, d\theta$$

Put $\sin\theta = t \Rightarrow \cos\theta\, d\theta = dt$

$$\therefore\quad I_2 = \int \frac{dt}{4+t^2-4t} = \int \frac{dt}{(t-2)^2}$$

$$= \frac{-1}{t-2} + c_2 = \frac{-1}{\sin\theta - 2} + c_2$$

$$\therefore\quad I = 3\log(\sin\theta - 2) - \frac{6}{\sin\theta - 2} + \frac{2}{\sin\theta - 2} + c,$$

where $c = 3c_1 - 2c_2$

$$= 3\log(\sin\theta - 2) - \frac{4}{\sin\theta - 2} + c$$

OR

Let $I = \int_0^{\pi} e^{2x}\cdot\sin\left(\frac{\pi}{4}+x\right)dx$

Put $\frac{\pi}{4} + x = t \Rightarrow x = t - \frac{\pi}{4} \Rightarrow dx = dt$

When $x = 0,\ t = \frac{\pi}{4}$ and when $x = \pi,\ t = \frac{5\pi}{4}$

$$\therefore\quad I = \int_{\pi/4}^{5\pi/4} e^{2\left(t-\frac{\pi}{4}\right)} \sin t\, dt = e^{-\pi/2}\int_{\pi/4}^{5\pi/4} e^{2t}\sin t\, dt$$

$$= e^{-\pi/2}\left[\left(\sin t\,\frac{e^{2t}}{2}\right)_{\pi/4}^{5\pi/4} - \int_{\pi/4}^{5\pi/4} \cos t\,\frac{e^{2t}}{2}dt\right]$$

$$= e^{-\pi/2}\left[\frac{1}{2}\left(e^{5\pi/2}\sin\frac{5\pi}{4} - e^{\pi/2}\sin\frac{\pi}{4}\right) - \left(\frac{e^{2t}}{4}\cos t\right)_{\pi/4}^{5\pi/4} - \int_{\pi/4}^{5\pi/4}\frac{e^{2t}}{4}\sin t\, dt\right]$$

$$= e^{-\pi/2}\left[\frac{1}{2}\left(\frac{-1}{\sqrt{2}}e^{5\pi/2} - \frac{1}{\sqrt{2}}e^{\pi/2}\right) - \frac{1}{4}\left(-\frac{1}{\sqrt{2}}e^{5\pi/2} - \frac{1}{\sqrt{2}}e^{\pi/2}\right)\right] - \frac{I}{4}$$

$$\Rightarrow\quad I + \frac{1}{4}I = -\frac{1}{2\sqrt{2}}\left[e^{2\pi}+1\right] + \frac{1}{4\sqrt{2}}(e^{2\pi}+1)$$

$$\Rightarrow\quad \frac{5}{4}I = \frac{(e^{2\pi}+1)}{2\sqrt{2}}\left[\frac{1}{2}-1\right] = -\frac{1}{4\sqrt{2}}(e^{2\pi}+1)$$

$$\Rightarrow\quad I = \frac{-1}{5\sqrt{2}}(1+e^{2\pi})$$

17. Let $I = \int \frac{\sqrt{x}}{\sqrt{a^3 - x^3}}dx$

Put $x^{3/2} = t$

$$\Rightarrow\quad \frac{3}{2}x^{1/2}dx = dt$$

$$\therefore\quad I = \frac{2}{3}\int \frac{dt}{\sqrt{a^3-t^2}} = \frac{2}{3}\int \frac{dt}{\sqrt{(a^{3/2})^2 - t^2}}$$

$$= \frac{2}{3}\left[\sin^{-1}\left(\frac{t}{a^{3/2}}\right)\right] + c = \frac{2}{3}\left[\sin^{-1}\left(\frac{x^{3/2}}{a^{3/2}}\right)\right] + c$$

$$= \frac{2}{3}\sin^{-1}\left(\frac{x}{a}\right)^{3/2} + c$$

18. Let $I = \int_{-1}^{2} |x^3 - x|\,dx$

$$|x^3 - x| = \begin{cases} x^3 - x, & x \in (-1, 0) \cup [1, 2) \\ -(x^3 - x), & x \in (0, 1) \end{cases}$$

$$\therefore\quad I = \int_{-1}^{0} |x^3-x|\,dx + \int_{0}^{1} |x^3-x|\,dx + \int_{1}^{2} |x^3-x|\,dx$$

$$= \int_{-1}^{0} (x^3-x)dx + \int_{0}^{1} -(x^3-x)dx + \int_{1}^{2} (x^3-x)dx$$

$$= \left(\frac{x^4}{4} - \frac{x^2}{2}\right)_{-1}^{0} + \left(\frac{-x^4}{4} + \frac{x^2}{2}\right)_{0}^{1} + \left(\frac{x^4}{4} - \frac{x^2}{2}\right)_{1}^{2}$$

$$= \left[0 - \left(\frac{1}{4} - \frac{1}{2}\right)\right] + \left(-\frac{1}{4} + \frac{1}{2} - 0\right) + \left[(4-2) - \left(\frac{1}{4} - \frac{1}{2}\right)\right]$$

$$= \frac{1}{4} + \frac{1}{4} + 2 + \frac{1}{4} = \frac{11}{4}$$

19. $(1 - y^2)(1 + \log x)\, dx + 2xy\, dy = 0$

$\Rightarrow\quad (1 - y^2)(1 + \log x)\, dx = -2xy\, dy$

$$\Rightarrow\quad \frac{(1+\log x)}{x}dx = -\frac{2y}{1-y^2}dy$$

On integrating both sides, we get

$$\int \frac{(1+\log x)}{x} dx = -\int \frac{2y}{1-y^2} dy$$

$$\Rightarrow \quad \frac{(1+\log x)^2}{2} = \log|1-y^2| + c$$

Now, $y = 0$ when $x = 1$

$$\Rightarrow \quad \frac{(1+\log 1)^2}{2} = \log(1) + c \Rightarrow c = \frac{1}{2}$$

$$\therefore \quad \frac{(1+\log x)^2}{2} = \log|1-y^2| + \frac{1}{2}$$

$\Rightarrow \quad (1 + \log x)^2 = 2 \log |1 - y^2| + 1$

20. General equation of a plane passing through $L(2, 2, 1)$ is $a(x - 2) + b(y - 2) + c(z - 1) = 0$ ...(i)

It will pass through $M(3, 0, 1)$ and $N(4, -1, 0)$ if

$a(3 - 2) + b(0 - 2) + c(1 - 1) = 0$

$\Rightarrow a - 2b + 0 \cdot c = 0$ ...(ii)

$a(4 - 2) + b(-1 - 2) + c(0 - 1) = 0$

$\Rightarrow 2a - 3b - c = 0$ ...(iii)

Solving (ii) and (iii), we get

$$\frac{a}{2+0} = \frac{b}{0+1} = \frac{c}{-3+4} \Rightarrow \frac{a}{2} = \frac{b}{1} = \frac{c}{1} = \lambda$$

$\Rightarrow \quad a = 2\lambda, b = \lambda, c = \lambda$

$\therefore \quad$ From (i), we get

$2\lambda(x - 2) + \lambda(y - 2) + \lambda(z - 1) = 0$

$\Rightarrow \quad 2x + y + z - 7 = 0$ ...(*)

Equation of line passing through $A(3, -4, -5)$ and $B(2, -3, 1)$ is $\dfrac{x-3}{2-3} = \dfrac{y+4}{-3+4} = \dfrac{z+5}{1+5}$

$$\Rightarrow \quad \frac{x-3}{-1} = \frac{y+4}{1} = \frac{z+5}{6} = r(\text{say})$$

$\Rightarrow \quad x = -r + 3, y = r - 4, z = 6r - 5$

Any point on the line AB is $P(-r + 3, r - 4, 6r - 5)$

$\because$ It lies on the plane (*)

$\therefore \quad 2(-r + 3) + (r - 4) + (6r - 5) - 7 = 0$

$\Rightarrow \quad 5r = 10 \Rightarrow r = 2$

So, coordinates of point P are $(1, -2, 7)$.

Let P divides the line segment AB in $k : 1$

$$\therefore \quad \left(\frac{3\times1+2\times k}{k+1}, \frac{1\times(-4)+k\times(-3)}{k+1}, \frac{-5\times1+k\times1}{k+1}\right)$$

$= (1, -2, 7)$

$$\Rightarrow \quad \frac{3+2k}{k+1} = 1 \Rightarrow 3 + 2k = k + 1 \Rightarrow k = -2$$

Hence, P divides the line segment AB in the ratio 2 : 1 externally.

21. Number of white balls = 3

Number of red balls = 6

Total number of balls = 9

Let X be the random variable denoting the number of red balls drawn.

$\therefore \quad X$ can take values 0, 1, 2, 3, 4

$P(X = 0)$ = Probability of getting no red ball in four draws

$$= \frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} = \frac{1}{81}$$

$P(X = 1)$ = Probability of getting one red ball in four draws

$$= 4 \times \left[\frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} \times \frac{2}{3}\right] = \frac{8}{81}$$

$P(X = 2)$ = Probability of getting two red balls in four draws

$$= 6 \times \left(\frac{1}{3} \times \frac{1}{3} \times \frac{2}{3} \times \frac{2}{3}\right) = \frac{24}{81}$$

$P(X = 3)$ = Probability of getting three red balls in four draws

$$= 4 \times \left(\frac{1}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3}\right) = \frac{32}{81}$$

$P(X = 4)$ = Probability of getting four red balls in four draws

$$= \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} = \frac{16}{81}$$

The probability distribution of X is

X	0	1	2	3	4
P(X)	1/81	8/81	24/81	32/81	16/81

$$\text{Mean} = \sum XP(X) = 0 \times \frac{1}{81} + 1 \times \frac{8}{81} + 2 \times \frac{24}{81} + 3 \times \frac{32}{81} + 4 \times \frac{16}{81}$$

$$= \frac{216}{81} = \frac{8}{3}$$

$$\text{Variance} = \sum X^2 P(X) - (\text{Mean})^2$$

$$= 0 \times \frac{1}{81} + 1 \times \frac{8}{81} + 4 \times \frac{24}{81} + 9 \times \frac{32}{81} + 16 \times \frac{16}{81} - \left(\frac{8}{3}\right)^2$$

$$= \frac{648}{81} - \frac{64}{9} = \frac{8}{9}$$

22. The given information can be represented in the tabular form as below:

Machines	Time required to produce		Max. machine hours available
	A	B	
First Machine	3	2	12
Second Machine	3	1	9
Profit (in ₹)	7	4	

Let the manufacturer produces x units of product A and y units of product B.

$\therefore$ $3x + 2y \le 12$ and $3x + y \le 9$

Let Z denote the total profit.

$\therefore$ $Z = 7x + 4y$

Clearly $x \ge 0$ and $y \ge 0$

Above L.P.P. can be stated mathematically as:

Maximize $Z = 7x + 4y$

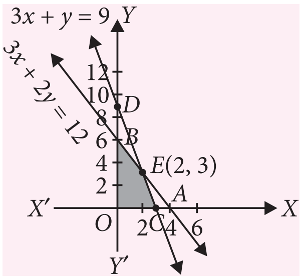
subject to $3x + 2y \le 12$,

$3x + y \le 9$ and $x, y \ge 0$

To solve graphically, we convert the inequations into equations

$3x + 2y = 12$, $3x + y = 9$, $x = 0$, $y = 0$

The line $3x + 2y = 12$ meets the coordinate axes at $A(4, 0)$ and $B(0, 6)$. Similarly $3x + y = 9$, meets the coordinate axes at $C(3, 0)$ and $D(0, 9)$



Coordinates of the corner points of the feasible region are $O(0, 0)$, $C(3, 0)$, $E(2, 3)$, $B(0, 6)$

Values of the objective function at corner points of the feasible region

Points	Value of objective function $Z = 7x + 4y$
$O(0, 0)$	$Z = 0$
$C(3, 0)$	$Z = 21$
$E(2, 3)$	$Z = 14 + 12 = 26$ (Max.)
$B(0, 6)$	$Z = 24$

$\therefore$ Z is maximum at $x = 2$, $y = 3$

For maximum profit he should manufacture 2 units of product A and 3 units of product B.

23. We have, $f : N \to S$, $f(x) = 9x^2 + 6x - 5$

Consider, $f(x_1) = f(x_2)$

$\Rightarrow$ $9x_1^2 + 6x_1 - 5 = 9x_2^2 + 6x_2 - 5$

$\Rightarrow$ $9(x_1^2 - x_2^2) + 6(x_1 - x_2) = 0$

$\Rightarrow$ $(x_1 - x_2)[9x_1 + 9x_2 + 6] = 0$

$\Rightarrow$ $x_1 = x_2$ $\qquad [\because x_1, x_2 \in N]$

$\Rightarrow$ f is one-one.

Let $y \in S$ be arbitrary number.

Consider, $y = f(x)$

$\Rightarrow$ $y = 9x^2 + 6x - 5 \Rightarrow y = (3x + 1)^2 - 6$

$$\Rightarrow \sqrt{y+6} = 3x+1 \Rightarrow x = \frac{\sqrt{y+6}-1}{3} \in N$$

$\Rightarrow$ $x = f^{-1}(y)$

Since, f is one-one and onto.

So, f is invertible.

$$\text{Also } f^{-1}(y) = \frac{\sqrt{y+6}-1}{3} \text{ or } f^{-1}(x) = \frac{\sqrt{x+6}-1}{3}$$

$$\text{Now, } f^{-1}(43) = \frac{\sqrt{49}-1}{3} = \frac{7-1}{3} = 2$$

$$\text{and } f^{-1}(163) = \frac{\sqrt{169}-1}{3} = \frac{13-1}{3} = 4$$

24. Let $\Delta = \begin{vmatrix} yz-x^2 & zx-y^2 & xy-z^2 \\ zx-y^2 & xy-z^2 & yz-x^2 \\ xy-z^2 & yz-x^2 & zx-y^2 \end{vmatrix}$

Applying $C_1 \to C_1 + C_2 + C_3$

$$\Delta = \begin{vmatrix} -(x^2+y^2+z^2-xy-yz-zx) & zx-y^2 & xy-z^2 \\ -(x^2+y^2+z^2-xy-yz-zx) & xy-z^2 & yz-x^2 \\ -(x^2+y^2+z^2-xy-yz-zx) & yz-x^2 & zx-y^2 \end{vmatrix}$$

$$\text{Applying } C_1 \to \frac{C_1}{-(x^2+y^2+z^2-xy-yz-zx)}$$

$$\Delta = -(x^2+y^2+z^2-xy-yz-zx) \begin{vmatrix} 1 & zx-y^2 & xy-z^2 \\ 1 & xy-z^2 & yz-x^2 \\ 1 & yz-x^2 & zx-y^2 \end{vmatrix}$$

Applying $R_1 \to R_1 - R_3$, $R_2 \to R_2 - R_3$

$$\Delta = -(x^2+y^2+z^2-xy-yz-zx) \begin{vmatrix} 0 & (x-y)(x+y+z) & (y-z)(x+y+z) \\ 0 & (x-z)(x+y+z) & (y-x)(x+y+z) \\ 1 & yz-x^2 & zx-y^2 \end{vmatrix}$$

$$\text{Applying } R_1 \to \frac{R_1}{(x+y+z)}, R_2 \to \frac{R_2}{(x+y+z)}$$

$$\Delta = -(x^2+y^2+z^2-xy-yz-xz)(x+y+z)^2 \begin{vmatrix} 0 & x-y & y-z \\ 0 & x-z & y-x \\ 1 & yz-x^2 & zx-y^2 \end{vmatrix}$$

$$\Rightarrow \quad \Delta = -(x+y+z)(x^3+y^3+z^3-3xyz) \begin{vmatrix} 0 & x-y & y-z \\ 0 & x-z & y-x \\ 1 & yz-x^2 & zx-y^2 \end{vmatrix}$$

Applying $R_1 \to R_1 - R_2$

$$\Delta = -(x+y+z)(x^3+y^3+z^3-3xyz) \begin{vmatrix} 0 & z-y & x-z \\ 0 & x-z & y-x \\ 1 & yz-x^2 & zx-y^2 \end{vmatrix}$$

Expanding along C_1, we get

$$\Delta = -(x+y+z)(x^3+y^3+z^3-3xyz)\,[0-0+\{(z-y)(y-x)-(x-z)^2\}]$$

$$=(x+y+z)(x^3+y^3+z^3-3xyz)\,(x^2+y^2+z^2-xy-yz-zx)$$

Hence, Δ is divisible by $(x + y + z)$ and quotient is $(x^3 + y^3 + z^3 - 3xyz)(x^2 + y^2 + z^2 - xy - yz - zx)$

OR

$$A = \begin{bmatrix} 8 & 4 & 3 \\ 2 & 1 & 1 \\ 1 & 2 & 2 \end{bmatrix}$$

Since, $AA^{-1} = I$

$$\Rightarrow \quad \begin{bmatrix} 8 & 4 & 3 \\ 2 & 1 & 1 \\ 1 & 2 & 2 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Applying $R_1 \leftrightarrow R_3$

$$\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 1 \\ 8 & 4 & 3 \end{bmatrix} A^{-1} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

Applying $R_2 \to R_2 - 2R_1$ and $R_3 \to R_3 - 8R_1$

$$\begin{bmatrix} 1 & 2 & 2 \\ 0 & -3 & -3 \\ 0 & -12 & -13 \end{bmatrix} A^{-1} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & -2 \\ 1 & 0 & -8 \end{bmatrix}$$

Applying $R_3 \to -R_3 + 4R_2$

$$\begin{bmatrix} 1 & 2 & 2 \\ 0 & -3 & -3 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & -2 \\ -1 & 4 & 0 \end{bmatrix}$$

Applying $R_2 \to \left(-\frac{1}{3}\right) R_2 - R_3$ and $R_1 \to R_1 - 2R_3$

$$\begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 2 & -8 & 1 \\ 1 & -13/3 & 2/3 \\ -1 & 4 & 0 \end{bmatrix}$$

Applying $R_1 \to R_1 - 2R_2$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 0 & 2/3 & -1/3 \\ 1 & -13/3 & 2/3 \\ -1 & 4 & 0 \end{bmatrix}$$

$$\text{So, } A^{-1} = \begin{bmatrix} 0 & 2/3 & -1/3 \\ 1 & -13/3 & 2/3 \\ -1 & 4 & 0 \end{bmatrix} \qquad \text{...(i)}$$

The given system of linear equations can be written as $AX = B$, where

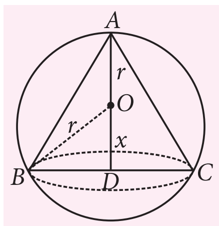
$$A = \begin{bmatrix} 8 & 4 & 3 \\ 2 & 1 & 1 \\ 1 & 2 & 2 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 19 \\ 5 \\ 7 \end{bmatrix}$$

$\therefore$ The solution of above equation is $X = A^{-1}B$

$$X = \begin{bmatrix} 0 & 2/3 & -1/3 \\ 1 & -13/3 & 2/3 \\ -1 & 4 & 0 \end{bmatrix} \begin{bmatrix} 19 \\ 5 \\ 7 \end{bmatrix} \qquad \text{[From (i)]}$$

$$= \begin{bmatrix} 0+10/3-7/3 \\ 19-65/3+14/3 \\ -19+20+0 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \Rightarrow x = 1, y = 2, z = 1$$

25. Let ABC be a cone of maximum volume inscribed in the sphere.



Let $OD = x$

$\Rightarrow \quad BD = \sqrt{r^2 - x^2}$

and $AD = AO + OD = r + x$

= altitude of cone.

Let V be the volume of cone

$$\Rightarrow \quad V = \frac{1}{3}\pi(BD)^2(AD) = \frac{1}{3}\pi(r^2 - x^2)(r+x)$$

$$\Rightarrow \quad \frac{dV}{dx} = \frac{1}{3}\pi\left[(r^2-x^2)+(r+x)(-2x)\right]$$

$$= \frac{\pi}{3}\left[r^2 - 3x^2 - 2rx\right]$$

$$\text{and } \frac{d^2V}{dx^2} = \frac{\pi}{3}[-6x - 2r]$$

For maximum or minimum, $\frac{dV}{dx} = 0$

$\Rightarrow \quad r^2 - 3x^2 - 2rx = 0 \Rightarrow r^2 - 3rx + rx - 3x^2 = 0$

$\Rightarrow \quad r(r-3x)+x(r-3x)=0 \Rightarrow (r-3x)(r+x)=0$

$\Rightarrow \quad r=3x \qquad [\because r+x \neq 0]$

$\Rightarrow \quad x=\frac{r}{3}$

Also, $\left(\frac{d^2V}{dx^2}\right)_{x=\frac{r}{3}}=\frac{\pi}{3}\left[-6\left(\frac{r}{3}\right)-2r\right]$

$=\frac{\pi}{3}[-2r-2r]=\frac{-4}{3}r\pi<0$

$\Rightarrow \quad V$ is maximum when $x=\frac{r}{3}$

and altitude of cone $= AD = r + x = r+\frac{r}{3}=\frac{4r}{3}$

Also, maximum volume of cone when $x=\frac{r}{3}$

$=\frac{1}{3}\pi\left(r^2-\frac{r^2}{9}\right)\left(r+\frac{r}{3}\right)=\frac{\pi}{3}\left(\frac{8}{9}r^2\right)\left(\frac{4}{3}r\right)$

$=\frac{8}{27}\left(\frac{4}{3}\pi r^3\right)=\frac{8}{27}$ (Volume of sphere)

OR

$f(x) = \sin 3x - \cos 3x \Rightarrow f'(x) = 3\cos 3x + 3\sin 3x$

$f'(x)= 0 \Rightarrow 3\cos 3x = -3\sin 3x$

$\Rightarrow \quad \cos 3x = -\sin 3x \Rightarrow \tan 3x = -1$

which gives $3x=\frac{3\pi}{4}$ or $\frac{7\pi}{4}$ or $\frac{11\pi}{4}$

$\Rightarrow \quad x=\frac{\pi}{4}$ or $\frac{7\pi}{12}$ or $\frac{11\pi}{12} \qquad [\because 0<x<\pi]$

The points $x=\frac{\pi}{4}$, $x=\frac{7\pi}{12}$ and $x=\frac{11\pi}{12}$ divides the interval into four disjoint intervals, namely

$\left(0,\frac{\pi}{4}\right),\left(\frac{\pi}{4},\frac{7\pi}{12}\right),\left(\frac{7\pi}{12},\frac{11\pi}{12}\right),\left(\frac{11\pi}{12},\pi\right)$

$\Rightarrow \quad f'(x)>0$ in $\left(0,\frac{\pi}{4}\right)$

or f is strictly increasing in $\left(0,\frac{\pi}{4}\right)$

$\Rightarrow \quad f'(x)<0$ in $\left(\frac{\pi}{4},\frac{7\pi}{12}\right)$

or f is strictly decreasing in $\left(\frac{\pi}{4},\frac{7\pi}{12}\right)$

$\Rightarrow \quad f'(x)>0$ in $\left(\frac{7\pi}{12},\frac{11\pi}{12}\right)$

or f is strictly increasing in $\left(\frac{7\pi}{12},\frac{11\pi}{12}\right)$

$\Rightarrow \quad f'(x)<0$ in $\left(\frac{11\pi}{12},\pi\right)$

or f is strictly decreasing in $\left(\frac{11\pi}{12},\pi\right)$

$\Rightarrow \quad f$ is strictly increasing in the intervals

$\left(0,\frac{\pi}{4}\right)\cup\left(\frac{7\pi}{12},\frac{11\pi}{12}\right)$

and f is strictly decreasing in the intervals

$\left(\frac{\pi}{4},\frac{7\pi}{12}\right)\cup\left(\frac{11\pi}{12},\pi\right)$

26. Let $R = \{(x, y) : x^2 + y^2 \le 2ax, y^2 \ge ax, x, y \ge 0\}$

$= \{(x, y) : x^2 + y^2 \le 2ax\} \cap \{(x, y) : y^2 \ge ax\}$

$\cap \{(x, y) : x \ge 0, y \ge 0\}$

$\Rightarrow \quad R = R_1 \cap R_2 \cap R_3$

where $R_1 = \{(x, y) : x^2 + y^2 \le 2ax\}$,

$R_2 = \{(x, y) : y^2 \ge ax\}$

$R_3 = \{(x, y) : x \ge 0, y \ge 0\}$

Region $R_1 : (x-a)^2 + y^2 = a^2$ represents a circle with centre at $(a, 0)$ and radius a.

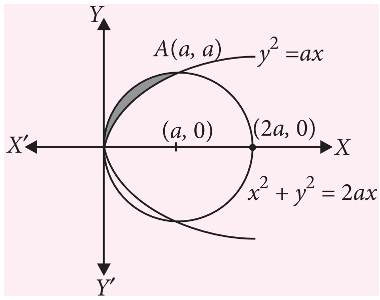
Region $R_2 : y^2 = ax$ represents a parabola with vertex at $(0, 0)$ and its axis along x-axis.

Region R_3 : $x \ge 0$, $y \ge 0$ represents the first quadrant.

$\Rightarrow \quad R = R_1 \cap R_2 \cap R_3$ is the shaded portion in the figure.

Since, given curves are $x^2 + y^2 = 2ax$ and $y^2 = ax$

So, point of intersection of the curves are $(0, 0)$ and (a, a)



$\therefore \quad$ Required area, $A=\int_0^a\left(\sqrt{a^2-(x-a)^2}-\sqrt{ax}\right)dx$

$=\left[\frac{1}{2}(x-a)\sqrt{a^2-(x-a)^2}+\frac{a^2}{2}\sin^{-1}\left(\frac{x-a}{a}\right)-\frac{2}{3}\sqrt{a}x^{3/2}\right]_0^a$

$A=\left\{\left(-\frac{2\sqrt{a}}{3}a^{3/2}\right)-\left(\frac{1}{2}a^2\sin^{-1}(-1)\right)\right\}$

$=\left\{-\frac{2a^2}{3}-\frac{a^2}{2}\left(-\frac{\pi}{2}\right)\right\}=\left(-\frac{2a^2}{3}+\frac{a^2\pi}{4}\right)$

$\Rightarrow \quad A=\left(\frac{\pi}{4}-\frac{2}{3}\right)a^2$ sq. units

MATHS MUSING

SOLUTION SET-159

1. (b) : $M=\sum_{s=1}^{10}\sum_{r=0}^{s-1}\left(2^s-2^r\right)$

$=\sum_{s=1}^{10}[s\cdot 2^s-(1+2+2^2+\ldots+2^{s-1})]=\sum_{s=1}^{10}\left((s-1)2^s+1\right)$

$= 4\,(1 + 2\cdot 2 + 3\cdot 2^2 + \ldots + 9\cdot 2^8) + 10$

$= 14 + 8\cdot 2^{11}$, summing A.G.P. $= 14 + 2^{14} = 16398$

2. (d) : H, T, X stand for head, tail, head or tail. The required sequence of atleast 8 consecutive heads is 8HXXXX, T8HXXX, XT8HXX, XXT8HX, XXXT8H.

Probability $=\frac{1}{2^8}+\frac{4}{2^9}=\frac{3}{2^8}$

3. (b): $\frac{m}{n}=\sum_{r=1}^{10}\frac{1}{r(r+1)(r+2)}=\frac{1}{2}\sum_{r=1}^{10}\left(\frac{1}{r}-\frac{2}{r+1}+\frac{1}{r+2}\right)$

$=\frac{1}{2}\left[1-1+\frac{1}{2}+\frac{1}{11}-\frac{2}{11}+\frac{1}{12}\right]=\frac{65}{264}$ $\therefore\ m+n=329$

4. (c) : $\alpha+\beta=-a,\ \alpha\beta=-\frac{1}{2a^2}\Rightarrow\ \alpha^2+\beta^2=a^2+\frac{1}{a^2}$

$\Rightarrow\ \alpha^4+\beta^4=a^4+2+\frac{1}{a^4}-\frac{1}{2a^4}=a^4+2+\frac{1}{2a^4}$

$=2+\left(a^2-\frac{1}{\sqrt{2}\cdot a^2}\right)^2+\sqrt{2}\geq 2+\sqrt{2}$

5. (b, c) : $\sin^{-1}t=t+\frac{1}{2}\cdot\frac{t^3}{3}+\frac{1\cdot 3}{2\cdot 4}\frac{t^5}{5}+\ldots$

$$I=\int_0^{\pi/2}\left(c\cos x+\frac{1}{2}\frac{c^3\cos^3 x}{3}+\frac{1}{2}\cdot\frac{3}{4}\cdot\frac{c^5\cos^5 x}{5}+\ldots\right)dx$$

$=c+\frac{c^3}{9}+\frac{c^5}{25}+\ldots$ $\therefore\ a_1+a_2+a_3=1+9+25=35$

6. (a) : $AF\parallel ED$ and $AE\parallel FD$

Now, in ΔABC and ΔEDC,

$\angle DEC=\angle BAC$, $\angle ACB$ is common.

$\Rightarrow\ \Delta ABC\sim\Delta EDC$

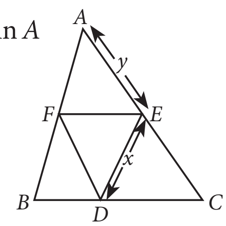
Now, $\frac{b-y}{b}=\frac{x}{c}\Rightarrow x=\frac{c}{b}(b-y)$

Now, S = Area of parallelogram $AFDE$ = 2 (Area of ΔAEF)

$\Rightarrow\ S=2\left(\frac{1}{2}xy\sin A\right)=\frac{c}{b}(b-y)\,y\sin A$

$\frac{dS}{dy}=\left(\frac{c}{b}\sin A\right)(b-2y)$

Sign scheme of $\frac{dS}{dy}$,



+ve $\quad b/2\quad$ −ve

Hence, S is maximum when $y=\frac{b}{2}$

$\therefore\ S_{\max}=\frac{c}{b}\left(\frac{b}{2}\right)\times\frac{b}{2}\sin A=\frac{1}{2}\left(\text{Area of }\Delta ABC\right)$

7. (b): The plane contains the point $C(1, 2, 3)$ and $A(0, -1, 1)$ and is parallel to the line with d.r.'s $-2, 2, 1$

$\therefore\ \begin{vmatrix} x & y+1 & z-1\\ -2 & 2 & 1\\ 1 & 3 & 2\end{vmatrix}=0\Rightarrow\ x+5y-8z+13=0$

The distance of $D(2, 0, 0)$ from the plane is $\frac{15}{\sqrt{90}}=\sqrt{\frac{5}{2}}$

8. (d): The foot of perpendicular from C on the line is $\left(\frac{-4}{3},\frac{1}{3},\frac{5}{3}\right)$

$\therefore$ The image of C in the line is $F\left(\frac{-11}{3},\frac{-4}{3},\frac{1}{3}\right)$

The distance of the point D from $F=\sqrt{34}$

9. (4): $P\left(\alpha,\frac{15}{8}\alpha\right)$ lies on $15x=8y$

$Q\left(\beta,\frac{3}{10}\beta\right)$ lies on $3x=10y$

The midpoint of PQ is $(8, 6)$

$\therefore\ \alpha+\beta=16,\frac{15}{8}\alpha+\frac{3}{10}\beta=12.$ Solving, $\alpha=\frac{32}{7},\beta=\frac{80}{7}$

$P=\left(\frac{32}{7},\frac{60}{7}\right),\ Q=\left(\frac{80}{7},\frac{24}{7}\right)\Rightarrow PQ=\frac{60}{7}=\frac{m}{n},m-8n=4$

10. (a) $\to$ (q); (b) $\to$ (t); (c) $\to$ (p); (d) $\to$ (r)

(a) x, y, z are in A.P. $\Rightarrow x+z=2y$, even

$\therefore$ Both x and z are even or both are odd

$\therefore$ The number of triples is $\binom{5}{2}+\binom{5}{2}=20$

(b) The number of triples is $\binom{10-3+1}{3}=\binom{8}{3}=56$

(c) 1 4 7 10
2 5 8
3 6 9

x^3+y^3 is divisible by 3 if $x+y$ is divisible by 3.

$\therefore$ x and y belong to 3rd row or one from 1st row and the second from 2nd row.

$\therefore$ The number of subsets is $\binom{3}{2}+\binom{4}{1}\binom{3}{1}=15$

(d) $x^2-y^2=(x+y)(x-y)$

$\therefore$ The number of subsets is $\binom{4}{2}+2\binom{3}{2}+\binom{4}{1}\binom{3}{1}=24$

JEE Advanced

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PRACTICE PAPER 2016

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SECTION-I

Single Correct Answer Type

1. Let $S \equiv \{a \in N, a \le 100\}$. If the equation $[\tan^2 x] - \tan x - a = 0$ has real roots then number of elements in S is, (where $[\cdot]$ represents the greatest integer function)

(a) 10 (b) 8 (c) 9 (d) 0

2. If $0 < x < 1$, the number of solutions of the equation $\tan^{-1}(x - 1) + \tan^{-1} x + \tan^{-1}(x + 1) = \tan^{-1} 3x$, is

(a) 0 (b) 1 (c) 2 (d) 3

3. ΔABC is inscribed in a unit circle. The three bisectors of angles A, B and C are extended to intersect the circle at A_1, B_1 and C_1 respectively. Then the value of

$$\frac{AA_1 \cos\frac{A}{2} + BB_1 \cos\frac{B}{2} + CC_1 \cos\frac{C}{2}}{\sin A + \sin B + \sin C} \text{ is}$$

(a) 2 (b) 4 (c) 6 (d) 8

4. The equation $2x = (2n + 1)\pi\,(1 - \cos x)$, (where n is a positive integer)

(a) has infinitely many real roots
(b) has exactly one real root
(c) has exactly $2n + 2$ real roots
(d) has exactly $2n + 3$ real roots

5. Let $F(x) = \sin x \int_0^x \cos t\, dt + 2\int_0^x t\, dt + \cos^2 x - x^2$. Then area bounded by $xF(x)$ and ordinate $x = 0$ and $x = 5$ with x-axis is

(a) 16 (b) $\frac{25}{2}$ (c) $\frac{35}{2}$ (d) 2

6. If $x = \sin\frac{2\pi}{7} + \sin\frac{4\pi}{7} + \sin\frac{8\pi}{7}$ and

$y = \cos\frac{2\pi}{7} + \cos\frac{4\pi}{7} + \cos\frac{8\pi}{7}$, then $x^2 + y^2$ is

(a) 1 (b) 2 (c) 3 (d) 4

7. The lengths of two opposite edges of a tetrahedron are a, b. Their shortest distance is d and the angle between them is θ. Then its volume is

(a) $\frac{1}{2}abd\sin\theta$ (b) $\frac{1}{3}abd\cos\theta$

(c) $\frac{1}{6}abd\sin\theta$ (d) $\frac{1}{6}abd\cos\theta$

8. Let S and I be the circumcentre and incentre of a triangle whose circumradius and inradius are 4 and 1 respectively. Then the length SI is

(a) $3\sqrt{2}$ units (b) $\sqrt{2}$ units
(c) 8 units (d) $2\sqrt{2}$ units

9. Let $A(1, 2)$, $B(3, 4)$ be two points and $C(x, y)$ be a point such that area of ΔABC is 3 sq.units and $(x - 1)(x - 3) + (y - 2)(y - 4) = 0$. Then maximum number of positions of C, in the xy plane is

(a) 2 (b) 4
(c) 8 (d) no such C exists

10. If A and B are foci of ellipse $(x - 2y + 3)^2 + (8x - 4y + 4)^2 = 20$ and P is any point on it, then $PA + PB$ is

(a) 2 (b) 4 (c) $\sqrt{2}$ (d) $2\sqrt{2}$

11. The ratio of the area enclosed by the locus of mid-point of PS and area of the ellipse where P is any point on the ellipse and S is the focus of the ellipse, is

(a) 1 : 2 (b) 1 : 3 (c) 1 : 5 (d) 1 : 4

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12. The quadrilateral formed by the lines $y = ax + c$, $y = ax + d$, $y = bx + c$ and $y = bx + d$ has area 18. The quadrilateral formed by the lines $y = ax + c$, $y = ax - d$, $y = bx + c$ and $y = bx - d$ has area 72. If a, b, c, d are positive integers, then the least possible value of the sum $a + b + c + d$ is

(a) 13 (b) 14 (c) 15 (d) 16

13. The number of ways of forming an arrangement of 5 letters from the letters of the word "IITJEE" is

(a) 60 (b) 96 (c) 120 (d) 180

14. Maximum value of $\log_5(3x + 4y)$, if $x^2 + y^2 = 25$ is

(a) 2 (b) 3 (c) 4 (d) 5

15. If the equation $x^n + a_1x^{n-1} + a_2x^{n-2} + ... + a_n = 5$, with integral co-efficients, has four distinct integral roots, then the number of integral roots of the equation $x^n + a_1x^{n-1} + a_2 x^{n-2} + ... + a_n = 7$ is

(a) 0 (b) 1 (c) 2 (d) 4

SECTION-II

Multiple Answer Correct Type

16. The solution set of $|\sin x| \le |\cos 2x|$ contains

(a) $\bigcup_{n\in I}\left\{\left[n\pi-\frac{\pi}{6}, n\pi+\frac{\pi}{6}\right]\right\}$

(b) $\bigcup_{n\in I}\left\{n\pi+\frac{\pi}{2}\right\}$

(c) $\bigcup_{n\in I}\left\{\left[n\pi-\frac{\pi}{8}, n\pi+\frac{\pi}{8}\right]\right\}$

(d) $\bigcup_{n\in I}\left\{\left[n\pi-\frac{\pi}{4}, n\pi+\frac{\pi}{4}\right]\right\}$

17. If the first and $(2n - 1)^{\text{th}}$ terms of an A.P., a G.P. and H.P. are equal and their n^{th} terms are p, q and s respectively, then which of the following options is/are correct?

(a) $p \ge q \ge s$ (b) $p + s = q$

(c) $ps = q^2$ (d) $p = q = s$

18. Given $|ax^2 + bx + c| \le |Ax^2 + Bx + C|$, $\forall\ x \in R$, $a, b, c, A, B, C \in R$ and $d = b^2 - 4ac > 0$ and $D = B^2 - 4AC > 0$. Then which of the following statements are true?

(a) $|a| \le |A|$ (b) $|d| > |D|$

(c) $|a| \ge |A|$ (d) None of these

19. If the equation $ax^2 + bx + c = 0$ and $bx^2 + cx + a = 0$ (a, b, c are unequal non-zero real) have a common root then $f(x) = bx^3 + cx^2 + ax - 5$ always passes through fixed point

(a) (1, –5) (b) (0, –5)

(c) (–1, –5) (d) (0, 5)

20. The value of $\sum_{k=0}^{7}\left[\frac{\binom{7}{k}}{\binom{14}{k}}\sum_{r=k}^{14}\binom{r}{k}\binom{14}{r}\right]$, where $\binom{n}{r}$ denotes ${}^nC_r$, is

(a) 6^7 (b) greater than 7^6

(c) 8^7 (d) greater than 7^8

21. If $\log_2(\log_{1/2}(\log_2(x))) = \log_3(\log_{1/3}(\log_3(y)))$ $= \log_5(\log_{1/5}(\log_5(z))) = 0$ for positive x, y and z, then which of the following is/are NOT true ?

(a) $z < x < y$ (b) $x < y < z$

(c) $y < z < x$ (d) $z < y < x$

22. $T_r = \dfrac{1}{r\sqrt{r+1}+(r+1)\sqrt{r}}$, then (here $r \in N$)

(a) $T_r > T_{r+1}$ (b) $T_r < T_{r+1}$

(c) $\sum_{r=1}^{99} T_r = \frac{9}{10}$ (d) $\sum_{r=1}^{n} T_r < 1$

23. Let A, B be two events such that $P(A\cup B) \ge \frac{3}{4}$ and $\frac{1}{8} \le P(A\cap B) \le \frac{3}{8}$. Then

(a) $P(A)+P(B) \le \frac{11}{8}$ (b) $P(A)\cdot P(B) \le \frac{3}{8}$

(c) $P(A)+P(B) \ge \frac{7}{8}$ (d) $P(A)\cdot P(B) > \frac{1}{2}$

24. Contents of the two urns are given in the table. A fair die is tossed. If the face 1,2,4 or 5 comes, a marble is drawn from the urn A, otherwise a marble is chosen from the urn B.

Urn	Red Marbles	White marbles	Blue marbles
A	5	3	8
B	3	5	0

Let

E_1 : event that a red marble is chosen

E_2 : event that a white marble is chosen

E_3 : event that a blue marble is chosen

Then

(a) The events E_1, E_2 and E_3 are equiprobable.

(b) $P(E_1)$, $P(E_2)$ and $P(E_3)$ are in A.P.

(c) If the marble drawn is red, the probability that it came from the urn A is 1/2.

(d) If the marble drawn is white, the probability that the face 5 appeared on the die is 3/32.

SECTION-III

Comprehension Type

Paragraph for Question No. 25 to 27

A right circular cone with radius R and height H contains a liquid which evaporates at a rate proportional to its surface area in contact with air (proportionality constant = $k > 0$). Suppose that $r(t)$ is the radius of liquid cone at time t.

25. The time after which the cone is empty is

(a) $H/2k$ (b) H/k
(c) $H/3k$ (d) $2H/k$

26. The radius of water cone at $t = 1$ is

(a) $R[1 - k/H]$ (b) $R[1 - H/k]$
(c) $R[1 + H/k]$ (d) $R[1 + k/H]$

27. The value of $\sum_{i=1}^{10} r(i)$ is equal to

(a) $10R\left[2-\frac{k}{H}\right]$ (b) $5R\left[2+\frac{11k}{H}\right]$
(c) $5R\left[2-\frac{11k}{H}\right]$ (d) $4R\left[2-\frac{11k}{H}\right]$

Paragraph for Question No. 28 to 30

If lines through $P(a, 2)$ meet the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$ at A and D and meet the axes at B and C, so that PA, PB, PC, PD are in G.P.

28. The interval for 'a' is

(a) $-6 \le a \le 6$ (b) $-6 < a < 6$
(c) $a > 6$ (d) $a \le 6$

29. The equation of all possible lines if $a = 10$ is

(a) $y - 2 = 2(x - 10)$ (b) $y - 10 = -2(x - 2)$
(c) $y + 2 = -2(x - 10)$ (d) $y + 10 = 2(x - 2)$

30. The slope of line AD is

(a) 1 or 1/9 (b) 2 or 2/9
(c) 3 or 3/9 (d) 4 or 5/9

Paragraph for Question No. 31 to 33

A circle C whose radius is 1 unit, touches the x-axis at point A. The centre Q of C lies in first quadrant. The tangent from origin O to the circle touches it at T and a point P lies on it such that ΔOAP is a right angled triangle at A and its perimeter is 8 units.

31. The length of QP is

(a) 1/2 (b) 4/3 (c) 5/3 (d) 5/2

32. Equation of circle C is

(a) $(x - 2)^2 + (y - 1)^2 = 1$
(b) $\{x-(2+\sqrt{3})\}^2+(y-1)^2=1$
(c) $(x-\sqrt{3})^2+(y-1)^2=1$
(d) none of these

33. Equation of tangent OT is

(a) $4x - 3y = 0$ (b) $x-\sqrt{3}y=0$
(c) $y-\sqrt{3}x=0$ (d) $x+\sqrt{3}y=0$

Paragraph for Question Nos. 34 to 36

For a finite set A, let $|A|$ denote the number of elements in the set A. Also let F denote the set of all functions $f: \{1, 2, 3, ..., n\} \to \{1, 2, ..., k\}$ $(n \ge 3, k \ge 2)$ satisfying $f(i) \ne f(i + 1)$ for every i, $1 \le i \le n - 1$.

34. $|F| =$

(a) $k^n(k - 1)$ (b) $k(k - 1)^{n-2}$
(c) $k^{n-1}(k - 1)$ (d) None of these

35. If $c(n, k)$ denote the number of functions in F satisfying $f(n) \ne f(1)$, then for $n \ge 4$, $c(n, k) =$

(a) $k(k - 1)^n - c(n - 1, k)$
(b) $k(k - 1)^n - c(n - 1, k - 1)$
(c) $k^{n-1}(k - 1) - c(n - 1, k)$
(d) None of these

36. For $n \ge k$, $c(n, k)$, where $c(n, k)$ has the same meaning as in given, equals

(a) $k^n + (-1)^n (k - 1)$
(b) $(k - 1)^n + (-1)^{n-1}(k - 1)$
(c) $(k - 1)^n + (-1)^n (k + 1)$
(d) None of these

SECTION-IV

Matrix Match Type

37. If $\vec{a}$ and $\vec{b}$ are two unit vectors inclined at angle α to each other, then

	Column I		Column II
(A)	$\lvert\vec{a}+\vec{b}\rvert<1$ if	(P)	$\frac{2\pi}{3}<\alpha\le\pi$
(B)	$\lvert\vec{a}-\vec{b}\rvert=\lvert\vec{a}+\vec{b}\rvert$ if	(Q)	$\frac{\pi}{2}<\alpha\le\pi$
(C)	$\lvert\vec{a}+\vec{b}\rvert<\sqrt{2}$ if	(R)	$\alpha=\frac{\pi}{2}$
(D)	$\lvert\vec{a}-\vec{b}\rvert<\sqrt{2}$ if	(S)	$0\le\alpha<\frac{\pi}{2}$

38. z_1, z_2, z_3 are vertices of a triangle. Match the condition in column I with type of triangle in column II.

	Column I		Column II
(A)	$z_1^2+z_2^2+z_3^2=z_2z_3+z_3z_1+z_1z_2$	(P)	Right angled
(B)	$\mathrm{Re}\left(\dfrac{z_3-z_1}{z_3-z_2}\right)=0$	(Q)	Obtuse angled
(C)	$\mathrm{Re}\left(\dfrac{z_3-z_1}{z_3-z_2}\right)<0$	(R)	Isosceles and right angled
(D)	$\dfrac{z_3-z_1}{z_3-z_2}=i$	(S)	Equilateral

SECTION-V
Integer Type

39. If $\vec{a}, \vec{b}, \vec{c}$ be non-coplanar unit vectors equally inclined to one another at an acute angle θ, and if $\vec{a}\times\vec{b}+\vec{b}\times\vec{c}=p\vec{a}+q\vec{b}+r\vec{c}$ $(p, q, r\in R)$ then $p-r=$

40. Let A be a point on the line $\vec{r}=(-3\hat{i}+6\hat{j}+3\hat{k})+t(2\hat{i}+3\hat{j}-2\hat{k})$ and B be a point on the line $\vec{r}=6\hat{j}+s(2\hat{i}+2\hat{j}-\hat{k})$. The least value of the distance AB is

41. The smallest positive value of x for which $\tan x^\circ = \tan(x^\circ+10^\circ)\tan(x^\circ+20^\circ)\tan(x^\circ+30^\circ)$ is

42. Differential equation, having solution $y=(\sin^{-1}x)^2+A(\cos^{-1}x)+B$, where A and B are arbitary constants is $(p-x^2)\dfrac{d^2y}{dx^2}-\dfrac{xdy}{dx}=q$ then $p+q=$

43. If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors such that $\vec{a}$ is perpendicular to plane of $\vec{b}$ and $\vec{c}$ and the angle between $\vec{b}$ and $\vec{c}$ is $\pi/3$ then $|\vec{a}+\vec{b}+\vec{c}|$ is

44. If the angle between the asymptotes of hyperbola $\dfrac{x^2}{a^2}-\dfrac{y^2}{b^2}=1$ is $\pi/3$. Then the eccentricity of conjugate hyperbola is

45. Let $X=\{1, 2, 3, ... 100\}$ and Y be a subset of X such that the sum of no two elements in Y is divisible by 7. If the maximum possible number of element in Y is $40+\lambda$ then λ is

46. Let A_n, $(n\in N)$ be a matrix of order $(2n-1)\times(2n-1)$, such that $a_{ij}=0\ \forall\ i\neq j$ and $a_{ij}=n^2+i+1-2n\ \forall\ i=j$ where a_{ij} denotes the element of i^{th} row and j^{th} column of A_n.
Let $T_n=(-1)^n\times$ (sum of all the elements of A_n).
Find the value of $\left[\dfrac{\sum_{n=1}^{102}T_n}{520200}\right]$, where $[\cdot]$ represents the greatest integer function.

47. Let α, β be roots of the quadratic equation $18x^2-9\pi x+\pi^2=0$ where $\alpha<\beta$. Also $f(x)=x^2$ and $g(x)=\cos x$. If the area bounded by the curve $y=(fog)(x)$, the vertical lines $x=\alpha$, $x=\beta$ and x-axis is π/λ, then find the sum of the digits in λ.

SOLUTIONS

1. **(c):** The equation $[\tan^2 x]-\tan x-a=0$ is true only if $\tan x$ is an integer. Since $[\tan^2 x]$ and a both are integer.

$\Rightarrow \tan x=\dfrac{1\pm\sqrt{4a+1}}{2}$ = both value are integers

$\Rightarrow 4a+1=(2k-1)^2 \Rightarrow k^2-k=a$ and $a\le 100$

$\Rightarrow$ There are 9 values of a satisfied.

2. **(b)** **:** The given equation can be written as
$\tan^{-1}(x-1)+\tan^{-1}(x+1)=\tan^{-1}3x-\tan^{-1}x$

$\Rightarrow \dfrac{2x}{2-x^2}=\dfrac{2x}{1+3x^2} \Rightarrow x+3x^3=2x-x^3$

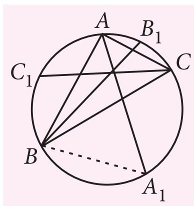
$\Rightarrow 4x^3-x=0 \Rightarrow x(4x^2-1)=0$

$\Rightarrow x=0,\ x=\pm\dfrac{1}{2}$. Thus $x=\dfrac{1}{2}$.

3. **(a)** **:** Join as shown in diagram then by sine rule for ΔABA_1

$$AA_1=2\sin\left(B+\frac{A}{2}\right)=2\cos\left(\frac{B}{2}-\frac{C}{2}\right)$$

Thus $AA_1\cos\dfrac{A}{2}=2\cos\left(\dfrac{B-C}{2}\right)\cos\dfrac{A}{2}=\sin C+\sin B$



Similarly, $BB_1\cos\dfrac{B}{2}=\sin A+\sin C$

and $CC_1\cos\dfrac{C}{2}=\sin A+\sin B$

So, $\dfrac{AA_1\cos\frac{A}{2}+BB_1\cos\frac{B}{2}+CC_1\cos\frac{C}{2}}{\sin A+\sin B+\sin C}=2$

4. **(c):** $\sin^2\left(\dfrac{x}{2}\right)=\dfrac{x}{(2n+1)\pi}$

The graph of $\sin^2\left(\dfrac{x}{2}\right)$ will be above the x–axis and will

be meeting the x-axis at 0, 2π, 4π, ... etc. It will attain maximum values at odd multiples of π *i.e.*, $\pi, 3\pi, ...(2n+1)\pi$.

The last point after which graph of $y=\dfrac{x}{(2n+1)\pi}$ will stop cutting will be $(2n + 1)\pi$.

Total intersection = $2(n + 1)$

5. (b) : $F(x)=\sin x\int_0^x \cos t\,dt+2\int_0^x t\,dt-x^2+\cos^2 x$

$$=\sin x(\sin t)_0^x+2\left(\frac{t^2}{2}\right)_0^x-x^2+\cos^2 x$$

$$= \sin^2 x + x^2 - x^2 + \cos^2 x = 1$$

Required Area $=\int_0^5 xF(x)dx=\int_0^5 (x)(1)dx=\left[\dfrac{x^2}{2}\right]_0^5=\dfrac{25}{2}$

6. (b) : $x^2+y^2=3+2\left(\cos\dfrac{2\pi}{7}+\cos\dfrac{4\pi}{7}+\cos\dfrac{6\pi}{7}\right)$

$$=3+2\left(-\frac{1}{2}\right)=2$$

7. (c): Considers $OABC$, $\overline{OA}=\vec{\alpha}$, $\overline{OB}=\vec{\beta}$, $\overline{OC}=\vec{\gamma}$

And OA, BC as a pair of opposite edges.

$$\left|\overline{OA}\right|=a,\ \left|\overline{BC}\right|=b$$

Equation of $\overrightarrow{OA}$ is $\vec{r}=\vec{O}+t\,\vec{\alpha}$

Equation of $\overrightarrow{BC}$ is $\vec{r}=\vec{\beta}+s(\vec{\beta}-\vec{\gamma})$

$$d=\left|\frac{-[\vec{\alpha}\,\vec{\beta}\,\vec{\gamma}]}{|\vec{\alpha}||\vec{\beta}-\vec{\gamma}|\sin\theta}\right|\Rightarrow V=\frac{abd\sin\theta}{6}$$

8. (d) : $SI=\sqrt{R^2-2Rr}=\sqrt{16-8}=2\sqrt{2}$ units

9. (d) : (x, y) lies on the circle, with AB as a diameter.

Area $(\Delta ABC) = 3$

$$\Rightarrow \frac{1}{2}(AB)(\text{altitude})=3$$

$$\Rightarrow \text{Altitude}=\frac{3}{\sqrt{2}} \Rightarrow \text{ no such "}C\text{" exists.}$$

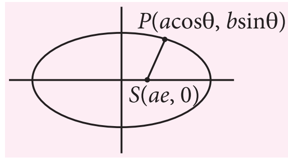
10. (b) : $\dfrac{\left(\dfrac{x-2y+3}{\sqrt{5}}\right)^2}{4}+\dfrac{\left(\dfrac{2x-y+1}{\sqrt{5}}\right)^2}{1/4}=1$

$a^2 = 4$, $PA + PB = 2a = 2 \times 2 = 4$

11. (d) : Let $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$ be an ellipse. Its focus is $(ae, 0)$ and area $= \pi ab$.

Let (h, k) be the mid-point of PS.

$$\therefore\ h=\frac{a\cos\theta+ae}{2}$$



$$\Rightarrow\ \cos\theta=\frac{2h-ae}{a}$$

and $k=\dfrac{b\sin\theta}{2} \Rightarrow \sin\theta=\dfrac{2k}{b}$.

$$\therefore\ \frac{(2h-ae)^2}{a^2}+\frac{4k^2}{b^2}=1 \Rightarrow \frac{4\left(h-\frac{ae}{2}\right)^2}{a^2}+\frac{4k^2}{b^2}=1$$

$$\Rightarrow \frac{\left(h-\frac{ae}{2}\right)^2}{\left(\frac{a}{2}\right)^2}+\frac{k^2}{\left(\frac{b}{2}\right)^2}=1$$

This is also an ellipse whose area $=\pi\dfrac{a}{2}.\dfrac{b}{2}=\dfrac{\pi ab}{4}$

$\therefore$ Required ratio = 1 : 4

12. (d) : $\dfrac{(c-d)^2}{|a-b|}=18$ and $\dfrac{(c+d)^2}{|a-b|}=72$

$a = 3$, $b = 1$, $d = 3$, $c = 9$ is a solution for which the minimum is attained.

13. (d) : Number of arrangements in which 2 are identical of one kind, two identical of another kind and one letter different from the remaining two letters is ${}^2C_1\times\dfrac{5!}{(2!)^2}=60.$

Number of arrangements in which 2 are identical of one kind and the rest are different is ${}^2C_1\times\dfrac{5!}{2!}=120$

14. (a): Since $x^2+y^2=25 \Rightarrow x=5\cos\theta$ and $y=5\sin\theta$

So, therefore, $\log_5(3x + 4y) = \log_5(15\cos\theta + 20\sin\theta)$

$\Rightarrow \{\log_5(3x + 4y)\}_{\max} = 2$

15. (a) : Let a, b, c, d be four distinct integral roots of $x^n + a_1x^{n-1} + ... + a_n = 5$

$\therefore\ f(x) = x^n + a_1x^{n-1} + ...\, a_n$

$$= (x - a)(x - b)(x - c)(x - d)\cdot g(x) = 5$$

$g(x)$ is of degree $(n - 4)$

Let $x = k$ be one integral root of $x^n + a_1x^{n-1} + ...\, a_n = 7$

So, $(k - a)(k - b)(k - c)(k - d)\cdot g(k) = 2$ which is not possible as $(k - a)(k - b)(k - c)(k - d)$ all distinct integers their product cannot be 2.

So no integral solutions.

16. (a, b) : $|\cos 2x|^2 \geq |\sin x|^2 \Rightarrow \cos^2 2x\geq\dfrac{1-\cos 2x}{2}$

$\Rightarrow$ $(\cos 2x + 1)(2\cos 2x - 1) \geq 0$

$\Rightarrow$ either $\cos 2x = -1$ or $\cos 2x \geq \dfrac{1}{2}$

$\Rightarrow$ either $2x \in 2n\pi + \pi$

or $\left[2n\pi - \dfrac{\pi}{3}, 2n\pi + \dfrac{\pi}{3}\right], n \in I$

17. (a, c): Let the first term be a and $(2n-1)^{\text{th}}$ term be b then

$$\Rightarrow \ p = a + (n-1)d = a + (n-1)\left(\frac{b-a}{2n-2}\right) = \frac{a+b}{2}$$

$$\Rightarrow \ q = a \cdot r^{n-1} = a\left(\frac{b}{a}\right)^{\frac{n-1}{2n-2}} = a\left(\frac{b}{a}\right)^{1/2} = \sqrt{ab}$$

$$\Rightarrow \ \frac{1}{s} = \frac{1}{a} + (n-1)\left(\frac{\frac{1}{b}-\frac{1}{a}}{2n-2}\right) = \frac{\frac{1}{a}+\frac{1}{b}}{2}$$

$\Rightarrow$ p, q, s are the A.M, G.M, H.M of a and b.

$\therefore$ $p \geq q \geq s$ and $ps = q^2$

18. (a) : Let α and β are the roots of

$Ax^2 + Bx + C = 0$

$\because$ $|ax^2 + bx + c| \leq |Ax^2 + Bx + C| \ \forall x \in R$

$\Rightarrow$ $ax^2 + bx + c = 0$ also has α, β as roots

$\Rightarrow$ $|ax^2 + bx + c| = |a||x - \alpha||x - \beta| \leq |A||x - \alpha||x - \beta|$

$\Rightarrow$ $|a| \leq |A|$

19. (a, b) : $ax^2 + bx + c = 0$ and $bx^2 + cx + a = 0$ have a common root.

$\Rightarrow$ $a + b + c = 0$

$f(x) = bx^3 + cx^2 + ax - 5$

$f(0) = -5$ and $f(1) = a + b + c - 5 = -5$

$\Rightarrow$ $f(x)$ will always pass through $(0, -5)$ and $(1, -5)$

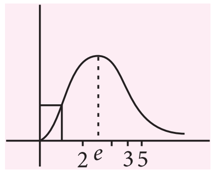
20. (a, b) : $\displaystyle\sum_{k=0}^{7}\left(\frac{^7C_k}{^{14}C_k}\sum_{r=k}^{14} {^rC_k}\,.\,{^{14}C_r}\right)$

$$= \sum_{k=0}^{7}\left(\frac{^7C_k}{\lfloor 14} \times \lfloor k \lfloor 14-k \sum_{r=k}^{14}\frac{\lfloor r}{\lfloor k \lfloor r-k} \cdot \frac{\lfloor 14}{\lfloor r \lfloor 14-r}\right)$$

$$= \sum_{k=0}^{7}\left({^7C_k}\sum_{r=k}^{14} {^{14-k}C_{r-k}}\right) = \sum_{k=0}^{7} {^7C_k}\,.\,2^{14-k}$$

$$= 2^{14}\sum_{k=0}^{7} {^7C_k}\left(\frac{1}{2}\right)^k = 2^{14}\,.\left(1+\frac{1}{2}\right)^7 = 6^7 > 7^6$$

21. (b, c, d) : Solving we get



$x = 2^{1/2}, y = 3^{1/3}, z = 5^{1/5}$

Using graph of $x^{1/x}$

$\Rightarrow$ $3^{1/3} > 5^{1/3}$

Also $2^{1/2} < 3^{1/3}$ as $2^3 < 3^2$

and $2^{1/2} > 5^{1/5}$ as $2^5 > 5^2$

$\Rightarrow$ $y > x > z$

Hence (b), (c), & (d) are NOT true.

22. (a, c, d) : $T_r = \dfrac{r(\sqrt{r+1}) - (r+1)\sqrt{r}}{r^2(r+1) - (r+1)^2 r}$

$$= \frac{r\sqrt{r+1} - (r+1)\sqrt{r}}{-r^2 - r} = \frac{(r+1)\sqrt{r}}{r(r+1)} - \frac{r\sqrt{r+1}}{r(r+1)}$$

$$= \frac{1}{\sqrt{r}} - \frac{1}{\sqrt{r+1}}$$

$$\Rightarrow \ \sum_{r=1}^{99} T_r = \frac{1}{1} - \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \ldots\ldots - \frac{1}{\sqrt{100}} = 1 - \frac{1}{\sqrt{100}} = \frac{9}{10}$$

Hence (a), (c) and (d) are correct.

23. (a, c): $\dfrac{3}{4} \leq P(A \cup B) \leq 1$

$$\Rightarrow \ \frac{3}{4} \leq P(A) + P(B) - P(A \cap B) \leq 1$$

$\therefore$ $P(A) + P(B) \geq \dfrac{3}{4} + P(A \cap B) \geq \dfrac{3}{4} + \dfrac{1}{8}$ and

$$P(A) + P(B) \leq 1 + P(A \cap B) \leq 1 + \frac{3}{8}$$

24. (a, b, d) :

(a) $P(E_1) = P(R) = \left(\dfrac{2}{3}\right)\left(\dfrac{5}{16}\right) + \left(\dfrac{1}{3}\right)\left(\dfrac{3}{8}\right) = \dfrac{10}{48} + \dfrac{6}{48} = \dfrac{1}{3}$

$P(E_2) = P(W) = \left(\dfrac{2}{3}\right)\left(\dfrac{3}{16}\right) + \left(\dfrac{1}{3}\right)\left(\dfrac{5}{8}\right) = \dfrac{6}{48} + \dfrac{10}{48} = \dfrac{1}{3}$

$P(E_3) = P(B) = \left(\dfrac{2}{3}\right)\left(\dfrac{8}{16}\right) = \dfrac{1}{3}$

(c) Let A : event that urn A is chosen

$$P(A/R) = \frac{P(A \cap R)}{P(R)} = \frac{\left(\frac{2}{3}\right)\left(\frac{5}{16}\right)}{\frac{1}{3}} = \left(\frac{10}{48}\right)(3) = \frac{5}{8}$$

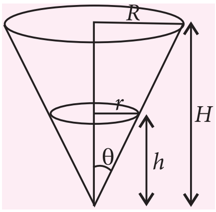
(d) $P(A/W) = \dfrac{P(A \cap W)}{P(W)} = \dfrac{\left(\frac{2}{3}\right)\left(\frac{3}{16}\right)}{\frac{1}{3}} = \left(\dfrac{6}{48}\right)(3) = \dfrac{3}{8}$

$$P(\text{face five}/W) = \left(\frac{3}{8}\right)\left(\frac{1}{4}\right) = \frac{3}{32}$$

(25-27) :

25. (b) **26. (a)** **27. (c)**

Let θ be the semi vertical angle of the cone so that $\tan\theta = R/H$.



Let the radius and height of water cone at time t be r and h respectively.

So, $\tan\theta = \dfrac{r}{h}$

If V is the volume of water and S is the surface area of the cone in contact with air at time t, then

$$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi r^3 \cot\theta \quad \text{and } S = \pi r^2$$

We are given that $\dfrac{dV}{dt} \propto S$

$$\Rightarrow \frac{dV}{dt} = -kS \ (\because V \text{ is decreasing})$$

$$\Rightarrow \frac{1}{3}\pi(3r^2)\cot\theta\frac{dr}{dt} = -k(\pi r^2) \Rightarrow \frac{dr}{dt} = -k\tan\theta$$

Integrating, we get $r = -(k\tan\theta)t + C$

When $t = 0$, $r = R$, $\therefore C = R$

Thus, $r = (-k\tan\theta)t + R$

When cone is empty, $r = 0$. If T is the time taken for the cone to be empty, then $0 = (-k\tan\theta)T + R$

$$\Rightarrow T = \frac{R}{k\tan\theta} = \frac{R}{k(R/H)} = \frac{H}{k}$$

Hence, cone will be empty in time H/k.

$$r(1) = -k\tan\theta + R = -k\frac{R}{H} + R = R\left(1 - \frac{k}{H}\right)$$

$$\sum_{i=1}^{10} r(i) = 10R - \frac{R}{H}k\sum_{i=1}^{10} i = 10R - \frac{Rk}{H}55$$

(28-30) :

28. (c) **29. (a)** **30. (b)**

Let equation of lines through $P(a, 2)$ be $x = a + r\cos\theta$ and $y = 2 + r\sin\theta$.

Lines meet the ellipse $\dfrac{x^2}{9} + \dfrac{y^2}{4} = 1$

or $4x^2 + 9y^2 = 36$ at A and D.

$\Rightarrow 4(a + r\cos\theta)^2 + 9(2 + r\sin\theta)^2 = 36$

So, $PA \cdot PD = \dfrac{4a^2}{4\cos^2\theta + 9\sin^2\theta}$

Since line meet the axes at B and C.

So, $PB \cdot PC = \dfrac{2a}{\sin\theta\cos\theta}$

Since, PA, PB, PC and PD are in G.P.

$\therefore PA \cdot PD = PB \cdot PC$

$$\Rightarrow \frac{4a^2}{4\cos^2\theta + 9\sin^2\theta} = \frac{2a}{\sin\theta\cos\theta}$$

$\Rightarrow 2a\sin 2\theta + 5\cos 2\theta = 13$

$$\Rightarrow -1 < \frac{13}{\sqrt{4a^2 + 25}} < 1 \Rightarrow a > 6 \text{ or } a < -6$$

For $a = 10$, $20\sin 2\theta + 5\cos 2\theta = 13$.

$\Rightarrow 9\tan^2\theta - 20\tan\theta + 4 = 0$.

$\Rightarrow \tan\theta = 2$ or $\tan\theta = 2/9$

Hence required equation is

$$y - 2 = 2(x - 10) \text{ and } y - 2 = \frac{2}{9}(x - 10)$$

(31-33) :

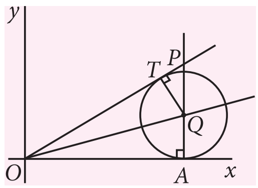
31. (c) **32. (a)** **33. (a)**

Given $QT = QA = 1$

Let $PQ = x$, then $PT = \sqrt{x^2 - 1}$

Then ΔTQP and ΔAPO are similar triangles

Then $OT = OA = \dfrac{x+1}{\sqrt{x^2-1}}$



$$\Rightarrow 1 + x + \frac{2(x+1)}{\sqrt{x^2-1}} + \sqrt{x^2-1} = 8$$

$$\Rightarrow x = \frac{5}{3}$$

$$AP = \frac{8}{3}, OP = \frac{10}{3}; \text{ and } OA = 2$$

$\therefore Q = (2, 1)$

Equation of the circle is $(x - 2)^2 + (y - 1)^2 = 1$

Coordinates of P are $\left(2, \dfrac{8}{3}\right)$

$\therefore$ Equation of OT is $4x - 3y = 0$.

34. (d) **35. (d)** **36. (d)**

37. (A - P), (B - R), (C - P, Q), (D - S)

If $|\vec{a} + \vec{b}| < 1$ then $(\vec{a} + \vec{b}) \cdot (\vec{a} + \vec{b}) < 1$

So $|\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a} \cdot \vec{b} < 1 \Rightarrow \vec{a} \cdot \vec{b} < -\dfrac{1}{2}$

$$\Rightarrow \cos\alpha < -\frac{1}{2} \Rightarrow \frac{2\pi}{3} < \alpha \leq \pi$$

If $|\vec{a}-\vec{b}|=|\vec{a}+\vec{b}|$ then $\vec{a}\cdot\vec{b}=0 \Rightarrow \alpha=\frac{\pi}{2}$

If $|\vec{a}+\vec{b}|<\sqrt{2}$ then $\cos\alpha<0$ which is true if $\frac{\pi}{2}<\alpha\le\pi$

If $|\vec{a}-\vec{b}|<\sqrt{2}$ then $\cos\alpha>0$ which is true if $0\le\alpha<\frac{\pi}{2}$

38. (A - S), (B - P), (C - Q), (D – R)

(A) $\frac{z_3-z_1}{z_2-z_1}=\frac{z_1-z_2}{z_3-z_2}=e^{i\frac{\pi}{3}}$

(B) $\operatorname{Re}\left(\frac{z_3-z_1}{z_3-z_2}\right)=0 \Rightarrow \operatorname{Arg}\left(\frac{z_3-z_1}{z_3-z_2}\right)=\pm\frac{\pi}{2}$

(C) $\operatorname{Re}\left(\frac{z_3-z_1}{z_3-z_2}\right)<0 \Rightarrow \operatorname{Arg}\left(\frac{z_3-z_1}{z_3-z_2}\right)>\frac{\pi}{2}$

(D) $\left(\frac{z_3-z_1}{z_3-z_2}\right)=i=e^{i\frac{\pi}{2}}$ and $|z_3-z_1|=|z_3-z_2|$

39. (0) : Taking dot product with

$\vec{a}=[\vec{a}\ \vec{b}\ \vec{c}]=p+q\cos\theta+r\cos\theta$ ... (1)

Taking dot product with

$\vec{c}=[\vec{a}\ \vec{b}\ \vec{c}]=p\cos\theta+q\cos\theta+r$ ... (2)

From (1) and (2), $p = r$

40. (3) : Let $A_0 = (-3, 6, 3)$, $B_0 = (0, 6, 0)$;

$\vec{c}=(2, 3, -2), \vec{d}=(2, 2, -1)$

Then $AB_{\min}=|\text{proj of } \overrightarrow{A_0B_0} \text{ on } \vec{c}\times\vec{d}\,| = 3$

41. (5) : $\frac{\tan x^\circ}{\tan(x^\circ+20^\circ)}=\tan(x^\circ+10^\circ)\tan(x^\circ+30^\circ)$

$\Rightarrow \frac{\sin(2x^\circ+20^\circ)}{\sin 20^\circ}=\frac{\cos 20^\circ}{\cos(2x^\circ+40^\circ)}$

$\Rightarrow \sin(4x^\circ + 60^\circ) - \sin 20^\circ = \sin 40^\circ$

$\Rightarrow \sin(4x^\circ + 60^\circ) = \sin 80^\circ$

$\Rightarrow$ Minimum positive value of $x = 5$

42. (3) : $\frac{dy}{dx}=\frac{2\sin^{-1}x}{\sqrt{1-x^2}}-\frac{A}{\sqrt{1-x^2}}$

$\Rightarrow (1 - x^2)y'' - xy' = 2$

43. (2) : $|\vec{a}|=|\vec{b}|=|\vec{c}|=1$ and $\vec{a}\cdot\vec{b}=0$ and $\vec{a}\cdot\vec{c}=0$

$\vec{b}\cdot\vec{c}=|\vec{b}||\vec{c}|\cos\frac{\pi}{3}=\frac{1}{2}.$

$\therefore\ |\vec{a}+\vec{b}+\vec{c}|^2=3+2\cdot0+2\cdot0+1=4$

$\therefore\ |\vec{a}+\vec{b}+\vec{c}|=2$

44. (2) : $2\tan^{-1}\left(\frac{b}{a}\right)=\frac{\pi}{3}\Rightarrow\frac{b}{a}=\frac{1}{\sqrt{3}}$

$\therefore\ e^2=1+\frac{1}{3}=\frac{4}{3}$

Now, $\frac{1}{e'^2}+\frac{1}{e^2}=1 \Rightarrow \frac{1}{e'^2}+\frac{3}{4}=1$

$\Rightarrow \frac{1}{e'^2}=\frac{1}{4} \Rightarrow e'=2$

45. (5) : Let Y_i be the subset of X such that

$Y_i = \{7m + i, m \in I\}$

$Y_0 = \{7, 14, ... 98\}, n(Y_0) = 14$

$Y_1 = \{1, 8, 15 ...99\}, n(Y_1) = 15$

$Y_2 = \{2, 9, 16 ... , 100\}, n(Y_2) = 15$

$Y_3 = \{3, 10, 17 .... 94\}, n(Y_3) = 14$

$Y_4 = \{4, 11, 18 ... 95\}, n(Y_4) = 14$

$Y_5 = \{5, 12, ... 96\}, n(Y_5) = 14$

$Y_6 = \{6, 13, ... 97\}, n(Y_6) = 14$

The largest Y will consist of (i) an element of Y_0 (ii) Y_1 (iii) Y_2 (iv) Y_3 or Y_4

$\Rightarrow$ The maximum possible number of elements in $Y = 1 + 15 + 15 + 14 = 45.$

46. (2) : $a_{ij} = 0\ \forall\ i \ne j$ and $a_{ij} = (n - 1)^2 + i\ \forall\ i = j$

Sum of all the element of $A_n=\sum_{i=1}^{2n-1}[(n-1)^2+i]$

$= (2n - 1)(n - 1)^2 + (2n - 1)n$

$= 2n^3 - 3n^2 + 3n - 1 = n^3 + (n - 1)^3$

So, $T_n = (-1)^n [n^3 + (n - 1)^3]$

$= (-1)^n n^3 - (-1)^{n-1} (n - 1)^3 = V_n - V_{n-1}$

$\Rightarrow \sum_{n=1}^{102}T_n=\sum_{n=1}^{102}(V_n-V_{n-1})=V_{102}-V_0=(102)^3$

So, $\left[\frac{\sum_{n=1}^{102}T_n}{520200}\right]=2$

47. (3) : $(fog)(x) = f(\cos x) = \cos^2 x$

α, β are the roots of $18x^2 - 9\pi x + \pi^2 = 0$

$\Rightarrow \alpha=\frac{\pi}{6}, \beta=\frac{\pi}{3}$

Area $=\int_{\pi/6}^{\pi/3}\cos^2 x\,dx=\frac{\pi}{12}$

PRACTICE PAPER 2016

JEE | OTHER PETs

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1. If $f(x)=\int_0^{g(x)}\frac{dt}{\sqrt{1+t^3}}$, $g(x)=\int_0^{\cos x}(1+\sin t^2)\,dt$ then the value of $f'(\pi/2)=$
(a) 1 (b) –1
(c) 0 (d) 1/2

2. A box contains 10 tickets numbered from 1 to 10. Two tickets are drawn one by one without replacement. The probability that the "difference between the first drawn ticket number and the second is not less than 4" is
(a) $\frac{7}{30}$ (b) $\frac{14}{30}$
(c) $\frac{11}{30}$ (d) $\frac{10}{30}$

3. The circle $x^2 + y^2 = 16$ touches the sides BC, CA and AB of ΔABC respectively at D, E and F. If the lengths BD, CE and AF are consecutive integers then the largest side of the triangle is equal to
(a) 13 (b) 14
(c) 15 (d) cannot be determined

4. If $\frac{dy}{dx}=\frac{(x+1)^2+y-3}{x+1}$ and $y(1) = 1$ then minimum value of $y(x)$ is
(a) 1/2 (b) 2/3
(c) 3/4 (d) 3/5

5. Consider two quadratic expressions $f(x) = ax^2 + bx + c$ and $g(x) = ax^2 + px + q$ $(a, b, c, p, q \in R, b \neq p)$ such that their discriminants are equal. If $f(x) = g(x)$ has a root $x = \alpha$ then
(a) α will be A.M. of the roots of $f(x) = 0$
(b) α will be A.M. of the roots of $g(x) = 0$
(c) α will be A.M. of the roots of $f(x) = 0$ or $g(x) = 0$
(d) α will be A.M. of the roots of $f(x) = 0$ and $g(x) = 0$

6. If range of the function $f(x)=\frac{2\sin^2 x+2\sin x+3}{\sin^2 x+\sin x+1}$ is $[a, b]$ then the value of $3a - 6b + 4$ is
(a) 0 (b) –3 (c) –6 (d) –9

7. The area bounded by the curve
$$y=\int_{1/8}^{\sin^2 x}(\sin^{-1}\sqrt{t})dt+\int_{1/8}^{\cos^2 x}(\cos^{-1}\sqrt{t})\,dt$$
$\left(0\le x\le\frac{\pi}{2}\right)$ and the curve satisfying the differential equation $y(x + y^3)dx = x(y^3 - x)dy$ passing through $(4, -2)$ is
(a) $\frac{1}{8}\left(\frac{3\pi}{16}\right)^4$ (b) $\frac{3}{8}\left(\frac{5\pi}{16}\right)^3$
(c) $\frac{1}{8}\left(\frac{3\pi}{16}\right)^3$ (d) $\frac{5}{8}\left(\frac{3\pi}{16}\right)^2$

8. Points A, B, C, D are $(0, 0), (2, 0), (2, 1), (0, 1)$. Then the minimum possible value of $PA + PB + PC + PD$ for all positions of P in the xy-plane is
(a) $\sqrt{5}$ (b) $2\sqrt{5}$ (c) $3\sqrt{5}$ (d) $4\sqrt{5}$

9. If $|z - i| \le 2$ and $z_0 = 5 + 3i$, the maximum value of $|iz + z_0|$ is
(a) $2+\sqrt{31}$ (b) $\sqrt{31}-2$
(c) 7 (d) none of these

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10. A bag contains $(2n + 1)$ coins. It is known that 'n' of these coins have a head on both sides, where as the remaining $(n + 1)$ coins are fair. A coin is picked up at random from the bag and tossed. If the probability that toss results in a head is $\frac{31}{42}$, then n is equals to

(a) 12 (b) 11 (c) 10 (d) 13

11. Number of permutations of 1, 2, 3, 4, 5, 6, 7, 8 and 9 taken all at a time are such that the digit

1 appearing somewhere to the left of 2
3 appearing to the left of 4 and
5 somewhere to the left of 6, is

(e.g. 815723946 would be one such permutation)

(a) 9·7! (b) 8! (c) 5!·4! (d) 8!·4!

12. A function $g(x)$ is continuous in $[0, \infty)$ satisfying $g(1) = 1$ and if $\int_0^x 2xg^2(t)dt = \left(\int_0^x 2g(x-t)dt\right)^2$ then $g(x)$ is

(a) $\sqrt{x}$ (b) $x^{1/\sqrt{2}}$ (c) x (d) $x^{1+\sqrt{2}}$

13. $2n$ balls (all distinct in size) are arranged in a row. First few of these balls are black rest all white, both odd in number. The probability that there is exactly one black ball in one of all possible arrangements, is

(a) $\frac{n}{2^{2n-1}}$ (b) $\frac{n}{2^{n-1}}$

(c) $\frac{n}{2^{2n-2}}$ (d) $\frac{n}{2^{n-2}}$

14. If $f(x) = |1 - x|$, then the points where $\sin^{-1}(f|x|)$ is non-differentiable, are

(a) $\{0, 1\}$ (b) $\{0, -1\}$
(c) $\{0, 1, -1\}$ (d) none of these

15. A variable straight line with slope m $(m \neq 0)$ intersects the hyperbola $xy = 1$ at two distinct points. Then the locus of the point which divides the line segment between these two points in the ratio 1 : 2 is

(a) an ellipse (b) a hyperbola
(c) a circle (d) a parabola

16. ABC is an actue angled triangle. In which quadrant of the complex plane does the complex number $(\cos B - \sin A) + i(\sin B - \cos A)$ lies?

(a) second
(b) third
(c) first
(d) it can lie in different quadrants in different cases.

17. OA is the perpendicular drawn from the centre 'O' of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, to the tangent at any point P on the ellipse. If the normal to the ellipse at the point P meets the X-axis at B, then $(OA)\cdot(PB)$ is

(a) a^2 (b) $a\sqrt{a^2+b^2}$
(c) b^2 (d) $b\sqrt{a^2+b^2}$

18. Assume that f is continuous on $[a, b]$, $a > 0$ and differentiable on an open interval (a, b). If $\frac{f(a)}{a}=\frac{f(b)}{b}$, then there exist $x_0 \in (a, b)$ such that

(a) $x_0 f'(x_0) = f(x_0)$
(b) $f'(x_0) + x_0 f(x_0) = 0$
(c) $x_0 f'(x_0) + f(x_0) = 0$
(d) $f'(x_0) = x_0^2 f(x_0)$

19. The values of α for which three distinct chords drawn from $(\alpha, 0)$ to the ellipse $x^2 + 2y^2 = 1$ are bisected by the parabola $y^2 = 4x$ is

(a) $(8, \infty)$ (b) $(0, -4+\sqrt{17})$
(c) $(0, 4+\sqrt{17})$ (d) $(8, 4+\sqrt{17})$

20. $\lim_{n\to\infty}[(2^n+1)(7^n+10^n)]^{1/n} =$

(a) $\frac{\lfloor 10}{\lfloor 6}$ (b) ${}^{10}C_5$ (c) ${}^{10}C_7$ (d) ${}^{6}C_3$

21. For a curve $y = f(x)$, $f''(x) = 4x$ at each point (x, y) on it and it crosses the x-axis at $(-2, 0)$ an angle of 45° with positive direction of x-axis. If $f(x) = ax^3 + bx^2 + cx + d$ then the value of $3(a + b + c + d)$ equals

(a) 45 (b) –15 (c) –45 (d) none of these

22. Let $f(x)$ be a +ve function differentiable on $[0, a]$ such that $f(0) = 1$ and $f(a) = 3^{1/6}$. If $f'(x) \geq (f(x))^4 + (f(x))^{-2}$, then maximum value of a is

(a) $\pi/12$ (b) $\pi/3$ (c) $\pi/6$ (d) $\pi/36$

23. Let $f(x) = \cos^2 x + \cos^2 2x + \cos^2 3x$. Number of values of $x \in [0, 2\pi]$ for which $f(x)$ equals the smallest +ve integer is

(a) 3 (b) 4 (c) 5 (d) 6

24. The largest interval for which the solution of the differential equation $\frac{dy}{dx}=\frac{1+y^2}{1+x^2}$, where $y(0) = 1$ holds good is

(a) $x \in R - \{1\}$ (b) $x \in (-1, 1)$
(c) $x \in (-\infty, 1)$ (d) $x \in (0, \infty)$

25. Let $f : R \to R$ be a differentiable function at $x = 0$ satisfying $f(0) = 0$ and $f'(0) = 1$, then the value of $\lim\limits_{x\to 0}\frac{1}{x}\sum\limits_{n=1}^{\infty}(-1)^n f\left(\frac{x}{n}\right)$ is

(a) 0 (b) $-\ln 2$ (c) 1 (d) e

26. The radius of the largest circle with centre at $(a, 0)$ $(a > 0)$ that can be inscribed in the ellipse $x^2 + 4y^2 = 16a^2$ is

(a) $\frac{11a}{\sqrt{3}}$ (b) $\sqrt{\frac{11}{3}}a$
(c) $\frac{\sqrt{11}}{3}a$ (d) $\frac{11}{3}a$

27. Let the base AB of a triangle ABC be fixed and the vertex C lie on a fixed circle of radius 'r'. Lines through A and B are drawn to intersect CB and CA respectively at E and F such that $CE : EB = 1 : 2$ and $CF : FA = 1 : 2$. If the point of intersection P of these lines lies on the median through C for all positions of 'C' then the locus of 'P' is

(a) a circle of radius $r/2$
(b) a circle of radius $2r$
(c) a parabola of latus rectum $4r$
(d) a rectangular hyperbola

28. The shape of surface of a curved mirror such that light from a source at origin will be reflected in a beam of rays parallel to x-axis is

(a) circle (b) parabola
(c) ellipse (d) hyperbola

29. Given $f(z)$ = the real part of a complex number z. For example, $f(3 - 4i) = 3$. If $a \in N$, $n \in N$ then the value of $\sum\limits_{n=1}^{6a}\log_2\left|f((1+i\sqrt{3})^n)\right|$ has the value equal to

(a) $18a^2 + 9a$ (b) $18a^2 + 7a$
(c) $18a^2 - 3a$ (d) $18a^2 - a$

30. The equation of the curve whose subnormal is equal to a constant a is

(a) $y = ax + b$ (b) $y^2 = 2ax + 2b$
(c) $ay^2 - x^3 = a$ (d) none of these

31. If $I_1 = \int_0^{\pi/2} f(\sin 2x)\sin x\,dx$ and $I_2 = \int_0^{\pi/4} f(\cos 2x)\cos x\,dx$ then $\frac{I_1}{I_2} =$

(a) 1 (b) 2 (c) $\sqrt{2}$ (d) $\frac{1}{\sqrt{2}}$

32. If an ellipse slides between two perpendicular straight lines, then the locus of its centre is

(a) a parabola (b) an ellipse
(c) a hyperbola (d) a circle

33. Let P and Q be the respective intersections of the internal and external angle bisectors of the triangle ABC at C and the side AB produced. If $CP = CQ$, then the value of $(a^2 + b^2)$ is (where a and b and R have their usual meanings for ΔABC)

(a) $2R^2$ (b) $2\sqrt{2}\,R^2$
(c) $4R^2$ (d) $4\sqrt{2}\,R^2$

34. Largest term in the expression of $\frac{1}{201}, \frac{4}{208}, \frac{9}{227}, \frac{16}{264}, ...$ is

(a) 6^{th} term (b) 7^{th} term
(c) 8^{th} term (d) 10^{th} term

35. The number of real roots of the equation $\sin^4 x + \cos^7 x = 1\ \forall\ x \in (-\pi, \pi)$

(a) 4 (b) 3 (c) 2 (d) 0

SOLUTIONS

1. (b) : $f'(x)=\frac{g'(x)}{\sqrt{1+g^3(x)}}$;

$g'(x)=[1+\sin(\cos^2 x)](-\sin x)$

$\therefore\quad f'(x)=\frac{[1+\sin(\cos^2 x)](-\sin x)}{\sqrt{1+g^3(x)}}$

$f'(\pi/2)=\frac{(1+0)(-1)}{\sqrt{1+0}}=-1$

2. (a) : 1 2 3 4 $\underline{5\ 6\ 7\ 8\ 9\ 10}$

If 1^{st} drawn is 5 then 2^{nd} drawn can be 1 only. If 1^{st} is 6 then 2^{nd} can be 1 or 2

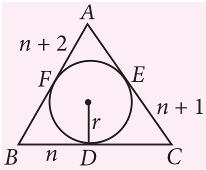
$\therefore\ P(E)=\frac{1}{10}\left[\frac{1}{9}+\frac{2}{9}+\frac{3}{9}+\frac{4}{9}+\frac{5}{9}+\frac{6}{9}\right]=\frac{1}{90}\left[\frac{6\cdot 7}{2}\right]=\frac{7}{30}$

3. (c): Let $BD = n$, $CE = n + 1$, $AF = n + 2$.

Then $BD = BF = n$, $CE = CD = n + 1$,

$AF = AE = n + 2$

$\therefore$ $a = BC = 2n + 1$, $b = 2n + 3$, $c = 2n + 2$



The largest side of the triangle is $2n + 3 = 15$

4. (c): $x + 1 = X$, $y - 3 = Y$

$$\frac{dY}{dX} - \frac{Y}{X} = X,\ I.F. = \frac{1}{X}$$

$\therefore$ Solution is $Y = X(X + c)$

$\Rightarrow$ $y = 3 + (x + 1)(x + 1 + c)$

$y(1) = 1 \Rightarrow c = -3$

$$\therefore\ y(x) = x^2 - x + 1 = \left(x - \frac{1}{2}\right)^2 + \frac{3}{4} \geq \frac{3}{4}$$

5. (d) : $a\alpha^2 + b\alpha + c = a\alpha^2 + p\alpha + q \Rightarrow \alpha = \dfrac{q - c}{b - p}$

$b^2 - 4ac = p^2 - 4aq \Rightarrow b^2 - p^2 = 4a(c - q)$

$$\Rightarrow\ b + p = \frac{4a(c - q)}{b - p} \Rightarrow b + p = -4a\alpha$$

$\therefore$ $\alpha = \dfrac{-(b + p)}{4a}$ which is A.M. of the roots of $f(x) = 0$ and $g(x) = 0$

6. (d) : $\dfrac{2\sin^2 x + 2\sin x + 3}{\sin^2 x + \sin x + 1} = 2 + \dfrac{1}{\left(\sin x + \frac{1}{2}\right)^2 + \frac{3}{4}}$

$$\therefore\ a = \frac{7}{3},\ b = \frac{10}{3}$$

7. (a) : $x(ydx + xdy) = y^3(xdy - ydx)$

$$\Rightarrow\ xd(xy) = y^3x^2\left(\frac{xdy - ydx}{x^2}\right)$$

$$\Rightarrow\ xd(xy) = x^2y^3d\left(\frac{y}{x}\right) \Rightarrow -\frac{1}{xy} = \frac{1}{2}\left(\frac{y}{x}\right)^2 + c$$

It passes through $(4, -2) \Rightarrow c = 0$

$y^3 + 2x = 0$, $y = (-2x)^{1/3}$

$$y = \int_{1/8}^{\sin^2 x} \sin^{-1}\sqrt{t}\,dt + \int_{1/8}^{\cos^2 x} \cos^{-1}\sqrt{t}\,dt$$

$y' = 2x\sin x \cos x - 2x\sin x \cos x = 0$

$\Rightarrow$ $y = c_1$. Put $\sin x = \cos x = \dfrac{1}{\sqrt{2}}$

$$y = \int_{1/8}^{1/2} \frac{\pi}{2}dx = \frac{3\pi}{16}$$

$$A = \int_0^{3\pi/16} xdy = \frac{1}{8}\left(\frac{3\pi}{16}\right)^4$$

8. (b) : $PA + PC$ is minimum when P is collinear with A and C. $PB + PD$ is minimum when P is collinear with B and D.

$\therefore$ $PA + PB + PC + PD$ is minimum when P is the point of intersection of diagonals AC and BD and its minimum value is $AC + BD$.

9. (c): $|iz + z_0| = |iz - i^2 + z_0 - 1| = |i(z - i) + 5 + 3i - 1|$

$= |i(z - i) + (4 + 3i)|$

$\leq |i(z - i)| + |4 + 3i|$

$\leq 1 \cdot 2 + 5 \leq 7$

10. (c): Let A_1 denote the event that a coin having heads on both sides is chosen, and A_2 denote the event that a fair coin is chosen. Let E denote the event that head occurs. Then

$$P(A_1) = \frac{n}{2n + 1},\ P(A_2) = \frac{n + 1}{2n + 1}$$

$P(E/A_1) = 1$, $P(E/A_2) = 1/2$

$P(E) = P(A_1 \cap E) + P(A_2 \cap E)$

$$\frac{31}{42} = \frac{n}{2n + 1}.1 + \frac{n + 1}{2n + 1}.\frac{1}{2}$$

$\therefore$ $n = 10$

11. (a) : Number of digits are 9

Select 2 places for the digit 1 and 2 in ${}^9C_2$ ways.

From the remaining 7 places select any two places for 3 and 4 in ${}^7C_2$ ways and from the remaining 5 places select any two for 5 and 6 in ${}^5C_2$ ways. Now, the remaining 3 digits can be filled in 3! ways.

$\therefore$ Total ways $= {}^9C_2 \cdot {}^7C_2 \cdot {}^5C_2 \cdot 3!$

$$= \frac{9!}{2! \cdot 7!} \cdot \frac{7!}{2! \cdot 5!} \cdot \frac{5!}{2! \cdot 3!} \cdot 3! = \frac{9!}{8} = \frac{9 \cdot 8 \cdot 7!}{8} = 9 \cdot 7!$$

12. (d) : $x\displaystyle\int_0^x g^2(t)dt = 2\left(\int_0^x g(t)dt\right)^2$ ... (1)

$$\left(\because \int_0^x g(t)dt = \int_0^x g(x - t)dt\right)$$

$$\int_0^x g^2(t)dt + xg^2(x) = 4g(x)\int_0^x g(t)dt$$

$\Rightarrow$ $x\displaystyle\int_0^x g^2(t)dt + x^2g^2(x) = 4xg(x)\int_0^x g(t)dt$ ... (2)

From (1) & (2)

$$2\left(\int_0^x g(t)dt\right)^2 - 4xg(x)\int_0^x g(t)dt + x^2g^2(x) = 0$$

$$\Rightarrow \int_0^x g(t)dt = \frac{2\pm\sqrt{2}}{2}xg(x)$$

$$\frac{g'(x)}{g(x)} = \frac{1\pm\sqrt{2}}{x} \Rightarrow g(x) = c\cdot x^{1\pm\sqrt{2}}$$

$g(1) = 1 \quad \Rightarrow \quad c = 1$

13. (c): Either first ball is black or first three balls are black or first five and so on... all being equally likely. Total probability of both balls being odd is

$$\frac{1}{k}\left[\frac{1}{1!(2n-1)!} + \frac{1}{3!(2n-3)!} + ...\right]$$

Where k is number of ways both can be odd.

$\therefore$ Probability of exactly one black ball is

$$\frac{\frac{1}{k}\left(\frac{1}{1!(2n-1)!}\right)}{\frac{1}{k}\left(\frac{1}{1!(2n-1)!} + \frac{1}{3!(2n-3)!} + ...\right)}$$

$$= \frac{2n}{{}^{2n}C_1 + {}^{2n}C_3 + ... + {}^{2n}C_{2n-1}} = \frac{2n}{2^{2n-1}} = \frac{n}{2^{2n-2}}$$

14. (c): Given that $f(x) = |1 - x|$

$$\Rightarrow \quad f(|x|) = \begin{cases} x-1, & x > 1 \\ 1-x, & 0 < x \le 1 \\ 1+x, & -1 \le x \le 0 \\ -x-1, & x < -1 \end{cases}$$

Clearly the domain of $\sin^{-1}(f|x|)$ is $[-2, 2]$.

$\therefore$ It is non-differentiable at the points $\{-1, 0, 1\}$.

15. (b) : Let the points of intersection be

$$\left(t_1, \frac{1}{t_1}\right) \text{ and } \left(t_2, \frac{1}{t_2}\right)$$

Given $m = -\frac{1}{t_1t_2}$ or $t_1t_2 = -\frac{1}{m}$

Also by section formula, $t_1 + 2t_2 = 3x$

$$2t_1 + t_2 = -\frac{3y}{m}$$

Solving for t_1, t_2 and eliminating them gives $2m^2x^2 + 5mxy + 2y^2 = m$ which is always a hyperbola as

$$\frac{25m^2}{4} - 4m^2 = \frac{9m^2}{4} > 0, \forall m \ne 0$$

16. (a) : We have $A + B > \frac{\pi}{2}$

$$\Rightarrow \quad 0 < \frac{\pi}{2} - A < B < \frac{\pi}{2}$$

$$\Rightarrow \quad \cos\left(\frac{\pi}{2} - A\right) > \cos B \Rightarrow \cos B < \sin A$$

$$\sin\left(\frac{\pi}{2} - A\right) < \sin B \quad \Rightarrow \quad \cos A < \sin B$$

$\therefore$ $\cos B - \sin A < 0$ and $\sin B - \cos A > 0$

$\therefore$ It lies in II quadrant.

17. (c): $P = (a\cos\theta, b\sin\theta)$

The tangent at P is $\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1$

$$OA = \frac{ab}{\sqrt{a^2\sin^2\theta + b^2\cos^2\theta}}$$

Normal at P is

$$\frac{ax}{\cos\theta} - \frac{by}{\sin\theta} = (a^2 - b^2)$$

$$B = \left(\frac{(a^2-b^2)}{a}\cos\theta, 0\right)$$

$$PB^2 = b^2\sin^2\theta + \left(a\cos\theta - \frac{(a^2-b^2)\cos\theta}{a}\right)^2$$

$$PB = \frac{b\sqrt{a^2\sin^2\theta + b^2\cos^2\theta}}{a}$$

$(OA)\cdot(PB) = b^2$

18. (a) : Consider a function $g(x) = \frac{f(x)}{x}$ as $f(x)$ and x are differentiable hence $g(x)$ is also differentiable

Now, $g(a) = \frac{f(a)}{a}$ and $g(b) = \frac{f(b)}{b}$

Since $\frac{f(a)}{a} = \frac{f(b)}{b}$

$\therefore$ $g(a) = g(b)$

Hence Rolle's theorem is applicable for $g(x)$

$\therefore$ There exists some $x_0 \in (a, b)$, where $g'(x) = 0$

But

$$g'(x) = \frac{x'f(x) - f(x)}{x^2}; \; g'(x_0) = \frac{x_0f'(x_0) - f(x_0)}{x_0^2} = 0$$

$\Rightarrow \; x_0 f'(x_0) = f(x_0)$

19. (d) : Let the middle point of chord be $(t^2, 2t)$. Mid point of chord must lie inside the ellipse.

$\Rightarrow\ t^4 + 8t^2 - 1 < 0 \Rightarrow t^2 \in (0, -4+\sqrt{17})$

Eqn. of chord is $t^2x + 4ty = ty + 8t^2$.

This passes through $(\alpha, 0)\ \Rightarrow\ t^2 = 0$ or $t^2 = \alpha - 8$.

$\therefore\ \alpha \in (8,\ 4+\sqrt{17})$

20. (d) : $\lim_{n\to\infty}\left[(20)^n\left(1+\frac{1}{2^n}\right)\left(1+\left(\frac{7}{10}\right)^n\right)\right]^{1/n} = 20 = {}^6C_3.$

21. (c): $f''(x) = 4x$

$f'(x) = 2x^2 + c$

$1 = +8 + c_1$ (slope is 45°)

$\Rightarrow\ c = -7$

$\therefore\ f'(x) = 2x^2 - 7$

$f(x) = +\frac{2x^3}{3} - 7x + c_2$ (passes through $(-2, 0)$)

$\Rightarrow\ c_2 = -\frac{26}{3}$

$f(x) = +\frac{2x^3}{3} - 7x - \frac{26}{3}$

and given $f(x) = ax^3 + bx^2 + cx + d$

$\Rightarrow\ a = +\frac{2}{3},\ b = 0,\ c = -7,\ d = -\frac{26}{3}$

$\therefore\ 3(a+b+c+d) = 3\left(\frac{2}{3} + 0 - 7 - \frac{26}{3}\right) = -45$

22. (d) : $f^2 f' \ge f^6 + 1$

$\Rightarrow\ 3 \le \frac{3f^2 f'}{(f^3)^2+1} \le \frac{d(f^3)}{(f^3)^2+1}$

Integrating on $[0, a]$, $3a \le \tan^{-1}\sqrt{3} - \tan^{-1}1$

$\Rightarrow\ a \le \frac{\pi}{36}$

23. (c): $f(x) = \cos^2 x + \cos^2 2x + \cos^2 3x$

$= 1 + \cos^2 x + \cos^2 2x - \sin^2 3x$

$= 1 + \cos^2 x + \cos 5x \cdot \cos x$

$= 1 + \cos x(\cos x + \cos 5x) = 1 + \cos x \cdot \cos 2x \cdot \cos 3x$

$\cos x \cdot \cos 2x \cdot \cos 3x = 0$

$\Rightarrow\ x = (2n-1)\frac{\pi}{2}$ or $x = (2n-1)\frac{\pi}{4}$ or $x = (2n-1)\frac{\pi}{6}$

$\Rightarrow\ x = \frac{\pi}{2}, \frac{\pi}{4}, \frac{3\pi}{4}, \frac{\pi}{6}, \frac{5\pi}{6}$

$\Rightarrow$ No. of values of $x = 5$

24. (c): $\tan^{-1}y = \tan^{-1}x + C$

$y(0) = 1 \Rightarrow \tan^{-1}1 = C \Rightarrow C = \pi/4$

$\therefore\ \tan^{-1}y = \tan^{-1}x + \frac{\pi}{4}$

$\Rightarrow\ \tan^{-1}x - \tan^{-1}y = \frac{-\pi}{4}$

And $y = \frac{x+1}{1-x}$

According to graph of $f(x) = \frac{x+1}{1-x}$ the largest interval is $(-\infty, 1)$.

25. (b) : $L = \lim_{x\to 0}\frac{1}{x}f\left(\frac{x}{n}\right) = \lim_{x\to 0}\frac{f\left(\frac{x}{n}\right) - f(0)}{x-0}$

and let $x/n = p$

$\therefore\ L = \frac{1}{n}\lim_{p\to 0}\frac{f(p)-f(0)}{p-0} = \frac{1}{n}f'(0) = \frac{1}{n}$

$\therefore\ \sum_{n=1}^{\infty}(-1)^n.\frac{1}{n} = -\ln 2$

26. (b) : Any point on given ellipse is $(4a\cos\theta, 2a\sin\theta)$

slope of tangent is $-\frac{1}{2}\frac{\cos\theta}{\sin\theta}$.

This is a tangent to circle with centre at (1,0) if

$-\frac{1}{2}\frac{\cos\theta}{\sin\theta} \times \frac{2\sin\theta}{4\cos\theta - 1} = -1 \Rightarrow \cos\theta = \frac{1}{3}$

$\therefore$ Radius of the required circle is

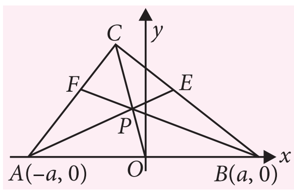
$a\sqrt{(4\cos\theta - 1)^2 + 4\sin^2\theta}$

$= a\sqrt{\left(\frac{4}{3}-1\right)^2 + 4\left(1-\frac{1}{9}\right)} = a\sqrt{\frac{11}{3}}$

27. (a) : Let A and B be $(-a, 0)$ and $(a, 0)$

Then by geometry we know $\frac{CP}{PO} = \frac{CF}{FA} + \frac{CE}{EB}$

$\therefore\ \frac{CP}{PO} = 1$



If $C(\alpha, \beta)$ lies on

$x^2 + y^2 + 2gx + 2fy + c = 0$

then

$\alpha = 2h$ and $\beta = 2k$

$\Rightarrow\ 4(h^2 + k^2 + gh + fk) + c = 0$

$\therefore$ Locus pt. $P(h, k)$ is $x^2 + y^2 + gx + fy + \frac{c}{4} = 0$

which is a circle of radius

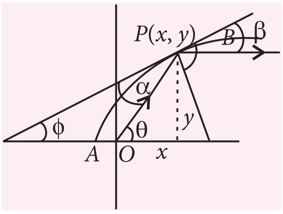
$= \sqrt{\left(\frac{g}{2}\right)^2 + \left(\frac{f}{2}\right)^2 - \frac{c}{4}} = \frac{1}{2}\sqrt{g^2 + f^2 - c} = \frac{r}{2}$

28. (b) : Let the shape of surface of a curved mirror be as shown .

By law of reflection, $\alpha = \beta$ but $\phi = \beta$

Also $\theta = \alpha + \phi = 2\beta = 2\phi$

Since $\tan\theta = \frac{y}{x}$, we get



$$\frac{y}{x} = \tan\theta = \tan 2\phi$$

$$= \frac{2\tan\phi}{1-\tan^2\phi} = \frac{2\frac{dy}{dx}}{1-\left(\frac{dy}{dx}\right)^2}$$

Solving gives

$$\frac{dy}{dx} = \frac{-x \pm \sqrt{x^2+y^2}}{y} \text{ or } \frac{xdx+ydy}{\sqrt{x^2+y^2}} = \pm dx$$

$$\Rightarrow \pm\sqrt{x^2+y^2} = x + c$$

Squaring gives $y^2 = 2cx + c^2$ which is a parabola .

29. (d) : $f((1+i\sqrt{3})^n) = \text{real part of } z = 2^n \cos\frac{n\pi}{3}$

$$\therefore \sum_{n=1}^{6a} \log_2\left|2^n \cos\frac{n\pi}{3}\right| = \sum_{n=1}^{6a}\left(n + \log_2\left|\cos\frac{n\pi}{3}\right|\right)$$

$$= \frac{6a(6a+1)}{2} + (-1-1+0-1-1+0+....)$$

$$= 3a(6a+1) - 4a = 18a^2 - a$$

30. (b) : $y\frac{dy}{dx} = a \Rightarrow ydy = adx$

$$\Rightarrow \frac{y^2}{2} = ax + b \Rightarrow y^2 = 2ax + 2b$$

31. (c): $I_1 = \int_0^{\pi/4} f(\sin 2x)\sin x dx + \int_{\pi/4}^{\pi/2} f(\sin 2x)\sin x dx$

Put $x = \frac{\pi}{4} - t$ in the first integral and

$x = \frac{\pi}{4} + t$ in the second integral

$$I_1 = \int_0^{\pi/4} f(\cos 2t)\sin\left(\frac{\pi}{4} - t\right)dt + \int_0^{\pi/4} f(\cos 2t)\sin\left(\frac{\pi}{4} + t\right)dt$$

$$= \sqrt{2}I_2$$

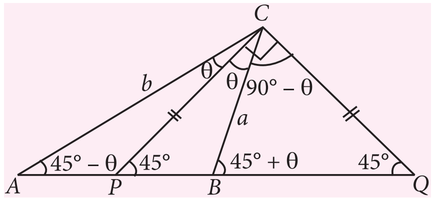
32. (d) : Let $2a$, $2b$ be the length of the major and minor axes respectively of the ellipse. If the ellipse slides between two perpendicular lines, the point of intersection P of these lines being the point of intersection of perpendicular tangents lies on the director circle of the ellipse.

$\Rightarrow$ Center C of ellipse is always at a distance of $\sqrt{a^2+b^2}$ from P.

33. (c): $a^2 + b^2 = 4R^2(\sin^2(45° - \theta) + \sin^2(135° - \theta))$

$= 4R^2(\sin^2(45° - \theta) + \cos^2(45° - \theta))$

$= 4R^2$



34. (b) : $T_n = \frac{x^2}{x^3+200} = f(x)$

$$f'(x) = \frac{x(400-x^3)}{(x^3+200)^2}$$

$f(x)$ is increasing in $0 < x < 400^{1/3}$ and decreasing in $x > 400^{1/3}$ largest term T_7.

$$T_7 = \frac{49}{543}$$

35. (b) : $\sin^4 x \le \sin^2 x$ ... (i)

And $\cos^7 x \le \cos^2 x$ ... (ii)

By adding (i) & (ii) we get

$\cos^7 x + \sin^4 x \le 1$

But $\cos^7 x + \sin^4 x = 1$ (Given)

And (i) and (ii) becomes equality only if

$\cos^2 x = \cos^7 x$ and $\sin^2 x = \sin^4 x$

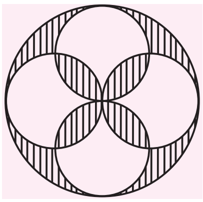
which are satisfied by $x = -\frac{\pi}{2}, \frac{\pi}{2}, 0$

OLYMPIAD CORNER

1. For a polynomial $P(x)$, define the difference of $P(x)$ on the interval $[a, b]$ ($[a, b)$, (a, b), $(a, b]$) as $P(b) - P(a)$.

   Prove that it is possible to dissect the interval $[0, 1]$ into a finite number of intervals and color them red and blue alternately such that, for every quadratic polynomial $P(x)$, the total difference of $P(x)$ on red intervals is equal to that of $P(x)$ on blue intervals. What about cubic polynomials?

2. In the pattern of circles shownthe largest circle has radius r. Find the area of the shaded part.



3. A graph with 8 vertices has no loops, multiple edges or 4-cycles. At most how many edges can it have?

4. Let $\frac{3}{4} < a < 1$. Prove that the equation
   $x^3(x+1) = (x+a)(2x+a)$
   has four distinct real solutions and find these solutions in explicit form.

5. Let $x_1, x_2, ..., x_n$ be variables, and let $y_1, y_2, ... y_{2^n-1}$ be the sums of non-empty subsets of x_i.
   Let $p_k(x_1,...,x_n)$ be the k^{th} elementary symmetric polynomial in the y_i (the sum of every product of k distinct y_i' s).
   For which k and n is every coefficient of p_k (as a polynomial in $x_1,..., x_n$) even?
   For example, if $n = 2$, then y_1, y_2, y_3 are $x_1, x_2, x_1 + x_2$ and
   $p_1 = y_1+y_2 + y_3 = 2x_1 + 2x_2$,
   $p_2 = y_1 y_2 + y_2y_3 + y_3y_1 = x_1^2 + x_2^2 + 3x_1x_2$,
   $p_3 = y_1 y_2 y_3 = x_1^2x_2 + x_1x_2^2$

SOLUTIONS

1. For an interval i, let $\Delta_i P$ denote the difference of polynomial P on i.
   For a positive real number c and a set $S \subseteq R$, let $S + c$ denote the set obtained by shifting S in the positive direction by c.
   We prove a more general result.
   Lemma :
   Let l be a positive real number, and let k be a positive integer. It is always possible to dissect interval $I_k = [0, 2^k l]$ into a finite number of intervals and color them red and blue alternatively such that, for every polynomial $P(x)$ with deg $P \le k$, the total difference of $P(x)$ on the red intervals is equal to that on the blue intervals.
   Proof :
   We induct on k.
   For $k = 1$, we can just use intervals $[0, l]$ and $(l, 2l]$. It is easy to see that a linear or constant polynomial has the same difference on the two intervals.
   Suppose that the statement is true for $k = n$, where n is a positive integer; that is, there exists a set R_n of red disjoint intervals and a set B_n, of blue disjoint intervals such that $R_n \cap B_n = \phi, R_n \cup B_n = I_n$, and, for any polynomials $P(x)$ with deg $P \le n$, the total differences of P on R_n is equal to that of P on B_n.
   Now consider polynomial $f(x)$ with deg $f \le n + 1$. Define $g(x) = f(x + 2^n l)$ and $h(x) = f(x) - g(x)$.
   Then deg $h \le n$. By the induction hypothesis,

$$\sum_{b\in B_n} \Delta_b h = \sum_{r\in R_n} \Delta_r h,$$

or $$\sum_{b\in B_n} \Delta_b f + \sum_{r\in R_n} \Delta_r g = \sum_{r\in R_n} \Delta_r f + \sum_{r\in B_n} \Delta_b g,$$

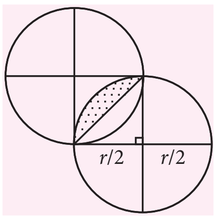
It follows that $\sum_{b\in B'_{n+1}} \Delta_b f = \sum_{r\in R'_{n+1}} \Delta_r f$,

where $R'_{n+1} = R_n \cup (B_n + 2^n l)$,
and $B'_{n+1} = B_n \cup (R_n + 2^n l)$.
(If R'_{n+1} and B'_{n+1} both contain the number $2^n l$, that number may be removed from one of them.)
It is clear that B'_{n+1} and R'_{n+1} form a dissection of I_{n+1} and, for any polynomial f with $\deg f \le n + 1$, the total difference of f on B'_{n+1} is equal to that of f on R'_{n+1}.
The only possible trouble left is that the colors in $B'_{n+1} \cup R'_{n+1}$ might not be alternating (which can happen at the end of the I_n and the beginning of $I_n + 2^n l$).
But note that if intervals $i_1 = [a_1, b_1]$ and $i_2 = [b_1, c_1]$ are in the same color, then $\Delta_{i_1} f + \Delta_{i_2} f = \Delta_{i_3} f$, where $i_3 = [a_1, c_1]$.
Thus, in $B'_{n+1} \cup R'_{n+1}$, we can simply put consecutive same color intervals into one bigger interval of the same color.
Thus, there exists a dissection $I_{n+1} = B_{n+1} \cup R_{n+1}$ such that, for every polynomial $f(x)$ with

$$\deg f \le n+1, \quad \sum_{b \in B_{n+1}} \Delta_b f = \sum_{r \in R_{n+1}} \Delta_r f$$

This completes the induction and the proof of the lemma.
Setting first $l = \frac{1}{4}$ and then $l = \frac{1}{8}$ in the lemma, it is clear that the answer to the given question is "yes."

2. Solution 1 :
In the adjacent figure, shaded area
= area of sector – area of triangle



$$= \frac{1}{2}\left(\frac{r}{2}\right)^2 \frac{\pi}{2} - \frac{1}{2}\left(\frac{r}{2}\right)^2$$

$$= \frac{r^2}{8}\left(\frac{\pi}{2} - 1\right)$$

The overlapping area of the two circles is therefore

$$= 2 \times \frac{r^2}{8}\left(\frac{\pi}{2} - 1\right) = \frac{r^2}{4}\left(\frac{\pi}{2} - 1\right).$$

Total area of the union of the four smaller circles in the given diagram = (sum of areas of circles) – 4 (overlapping area of two circles)

$$= 4\left(\frac{\pi r^2}{4}\right) - 4\left(\frac{\pi r^2}{8} - \frac{r^2}{4}\right) = \frac{\pi r^2}{2} + r^2$$

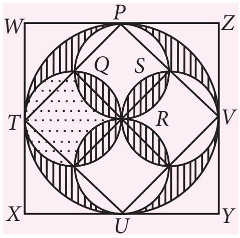
Therefore, sum of the outer shaded areas in the given diagram = (area of large circle) – (area of union of four smaller circles)

$$= \pi r^2 - \left(\frac{\pi r^2}{2} + r^2\right) = \frac{\pi r^2}{2} - r^2.$$

Thus the total area of the shaded region in the given

$$\text{diagram} = \frac{\pi r^2}{2} - r^2 + 4\left(\frac{\pi r^2}{8} + \frac{r^2}{4}\right) = (\pi - 2)r^2,$$

Solution 2 :
The solution can be shortened using various constructions. For example, construct the squares $RQRS$, $PTUV$ and $WXYZ$.



Therefore required area
= (area of larger circle) – 4 (dotted area)
= (area of larger circle) – 4 (area of square $PQRS$)
= (area of larger circle) – (area of square $PTUV$)
= (area of larger circle) $-\frac{1}{2}$ (area of square $WXYZ$)

$$= \pi r^2 - \frac{1}{2}(2r)^2$$

$$= (\pi - 2)r^2$$

3. Solution 1 :
Figure 1 shows that such a graph can have as many as 11 edges. Suppose there exists such a graph with 12 edges. Let A be the vertex with maximum degree n. Then $n \ge 3$. Let A be joined to $B_1, B_2, ..., B_n$, and let the remaining vertices be $C_1, C_2, ..., C_{7-n}$. If B_i or C_i is joined to both B_j and B_k, the three will form a 4–cycle with A. Hence there are at most $\left\lfloor \frac{n}{2} \right\rfloor$ edges of the type $B_i B_j$, and $7 - n$ edges of the type $B_i C_j$. Clearly, the number of edges of the type $C_i C_j$ is at most $\binom{7-n}{2}$.

Hence the total number of edges is at most

$$f(n) = n + \left\lfloor \frac{n}{2} \right\rfloor + (7-n) + \binom{7-n}{2}$$

Now $f(5) = f(6) = f(7) = 10 < 12$
while $f(4) = 12$. If $n = 4$, there are exactly 2, 3 and 3 edges of the types $B_i B_j$, $B_i C_j$ and $C_i C_j$ respectively,

we may assume that B_1 is joined to B_2, B_3 to B_4 and C_1, C_2 and C_3 to one another.
None of the B's can be joined to two of the C's as otherwise it will form a 4-cycle with the three C's. Hence three B's are joined to different C's. However, either B_1 and B_2 or B_4 will form a 4-cycle with two C's. Finally, for $n = 3$ all vertices are of degree 3. Although $f(3) = 14$, there can be at most 4 edges of the type $C_i\,C_j$. Hence we have exactly 1, 3 and 4 edges of the types $B_i\,B_j$, $B_i\,C_j$ and $C_i\,C_j$, respectively. Moreover one of the C's is joined to the other three. Hence another of the C's is joined to two of the B's thereby forming a 4-scycle with A.

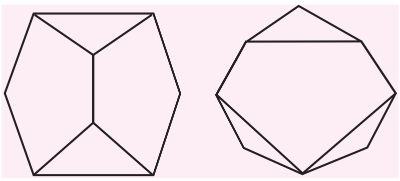


Figure 1 Figure 2

Solution 2 :
Figure 2 shows that such a graph can have as many as 11 edges. We claim that if there are only 7 vertices, then the total number of edges is at most 9. By the argument in the First Solution, it is at most

$$g(n) = n + \left\lfloor \frac{n}{2} \right\rfloor + (6-n) + \binom{6-n}{2}.$$

Now g(4) = g(6) = 9, g(5) = 8 while g(3) = 10. If $n = 3$, and the graph has 10 edges, we will have exactly 1, 3 and 3 edges of the types $B_i\,B_j$, $B_i\,C_j$ and $C_i\,C_j$ respectively. However, the two B's which are joined will form a 4 – cycle with two of the C's. This justifies our claim. Suppose there is a graph with 8 vertices and 12 edges, having no loops, multiple edges or 4 – cycles. If we delete any vertex, the resulting graph will have at most 9 edges by the claim. Hence each vertex is of degree at least 3, and it follows that every vertex is of degree 3. We can complete the argument as in the last case of the First Solution.

4. Look at the given equation as a quadratic equation in a:

$a^2 + 3xa + 2x^2 - x^3 - x^4 = 0$

The discriminant of this equation is

$9x^2 - 8x^2 + 4x^3 + 4x^4 = (x + 2x^2)^2$

Thus $a = \dfrac{-3x \pm (x+2x^2)}{2}$.

The first choice $a = -x + x^2$ yields the quadratic equation $x^2 - x - a = 0$, whose solutions are

$$x = \frac{(1 \pm \sqrt{1+4a})}{2}$$

The second choice $a = -2x - x^2$ yields the quadratic equation $x^2 + 2x + a = 0$,
whose solutions are $-1 \pm \sqrt{1-a}$.
The inequalities

$$-1-\sqrt{1-a} < -1+\sqrt{1-a} < \frac{1-\sqrt{1+4a}}{2} < \frac{1+\sqrt{1+4a}}{2}$$

show that the four solutions are distinct.

Indeed $-1+\sqrt{1-a} < \dfrac{1-\sqrt{1+4a}}{2}$

reduces to $2\sqrt{1-a} < 3 - \sqrt{1+4a}$

which is equivalent to $6\sqrt{1+4a} < 6 + 8a$,

or $3a < 4a^2$, which is evident.

5. We say a polynomial p_k is even if every coefficient of p_k is even.
Otherwise, we say p_k is not even
For any fixed positive integer n, we say a non-negative integer k is bad for n if $k = 2^n - 2^j$ for some non-negative integer j.
We will show by induction on n that $p_k\,(x_1, x_2, ..., x_n)$ is not even if and only if k is bad for n.
For $n = 1$, $p_1(x_1) = x_1$ is not even and $k = 1$ is bad for $n = 1$ as $k = 1 = 2^1 - 2^0 = 2^n - 2^0$
Suppose that the claim is true for a certain n.
We now consider $p_k\,(x_1, x_2, ..., x_{n+1})$.
Let $\sigma_k(y_1, y_2, ..., y_s)$ be the k^{th} elementary symmetric polynomial.
We have the following useful, but easy to prove, facts:

1. $\sigma_k\,(y_1, y_2, ..., y_s, 0) = \sigma_k\,(y_1, y_2, ..., y_s)$;
2. For all $1 \le r \le s$,
$$\sigma_k(y_1 ... y_s) = \sum_{i+j=k} [\sigma_i(y_i, ..., y_r)\sigma_j(y_{r+1}, ..., y_s)];$$
3. $\sigma_k\,(x + y_1, x + y_2, ..., x + y_s)$
$$= \sum_{i_1<i_2<...<i_k} (x+y_{i_1})(x+y_{i_2})...(x+y_{i_k})$$
$$= \sum_{i_1<i_2<...<i_k} \sum_{r=0}^{k} \sum_{\substack{s_1<s_2<...<s_r \\ \{s_1,...,s_r\}\subseteq\{i_1,...,i_k\}}} y_{s_1} y_{s_2} ... y_{s_r} x^{k-r}$$

$$= \sum_{r=0}^{k} \binom{s-r}{k-r} \sigma_r(y_1,\dots, y_s)x^{k-r}.$$

Hence $p_k\ (x_1, x_2,\dots,x_{n+1})$

$$= \sum_{i+j=k} [p_i(x_1,\dots,x_n)$$

$$\sigma_j(x_{n+1}, x_1+x_{n+1},\dots., x_1+x_2+\dots+x_{n+1})$$

$$= \sum_{i+j=k} \sum_{r=0}^{j} \binom{2^n-r}{j-r} p_i(x_1,\dots,x_n)p_r(x_1,\dots,x_n)x_{n+1}^{j-r}$$

By the induction hypothesis, every term of $p_r\ (x_1,x_2\dots,x_n)$ is even unless $r = 2^n - 2^t$, for some $0 \le t \le n$.

For such r, note that $\binom{2^n-r}{j-r}\binom{2^t}{j-r}$

is even unless $j - r = 0$ or $j - r = 2^t$.

Therefore, taking coefficients modulo 2,

$p_k(x_1, x_2,\dots, x_{n+1})$

$$\equiv \sum_{i+j=k} p_i(x_1,x_2,\dots,x_n)p_j(x_1,x_2,\dots x_n)$$

$$+\sum_{t-0}^{n} p_{k-2^n}(x_1,x_2,\dots,x_n)p_{2^n-2^t}(x_1,x_2,\dots x_n)x^{2^t}$$

By the induction hypothesis, the terms in the first sum are even unless $k - 2^n = 2^n - 2^u$ for some $0 \le u \le n$, that is $k = 2^{n+1} - 2^u$.

In the second sum, every term appears twice except the term $p_{k/2}(x_1,x_2,\dots,x_n)^2$, for k even.

By the induction hypothesis, this term is even unless $k/2 = 2^n - 2^u$, for some $0 \le v \le n$, that is

$$k = 2^{n+1} - 2^{v+1}.$$

It follows that $p_k(x_1, x_2,\dots,x_{n+1})$is even unless $k = 2^{n+1} - 2^w$ for some $0< w < n+1$,*i.e.*, k is bad for $n+1$;

Furthermore, note that the odd coefficients in

$$p_k(x_1,x_2,\dots,x_{n+1})$$

occur for different powers of x_{n+1}.

Therefore, the condition that k is bad for $n+1$ is also sufficient for $p_k(x_1, x_2,\dots,x_{n+1})$ to be odd.

Our induction is complete. ■■

PRACTICE PAPER 2016

JEE | OTHER PETs

1. If $a(l+m)^2+2blm+c=0$ and $a(l+n)^2+2bln+c=0$, $(a \neq 0)$ then
(a) $mn = l^2 + c/a$ (b) $mn = l^2$
(c) $mn = -l^2$ (d) none of these

2. If t_n denotes the n^{th} term of an A.P. and $t_p = 1/q$ and $t_q = 1/p$, then which of the following is necessarily a root of the equation $2\sin^{-1} x - \pi = 0$ is
(a) t_p (b) t_q (c) t_{pq} (d) t_{p+q}

3. If x and y are positive integers less than 15 and $(x-8)(x-10) = 2^y$, the number of ordered pair (x, y) satisfying the above equation is
(a) 1 (b) 2 (c) 0 (d) 4

4. The real values of 't' for which $\sin\alpha(\sin\alpha + \cos\alpha) = [t]$ (where $[\cdot]$ denotes greatest integral function) is
(a) $[0, 2)$ (b) $[0, 1] \cup [2, 3)$
(c) $[-1, 1) \cup [1, 2)$ (d) none of these

5. If $|z| = 1/2$ where z is complex number, then the complex number $(-1 + 4z)$ lies on a circle of radius
(a) 1/2 (b) 1 (c) 2 (d) 4

6. If a, b, c are positive numbers in G.P. and $\ln a - \ln 2b$, $\ln 2b - \ln 3c$ and $\ln 3c - \ln a$ are in A.P., then a, b, c are representing the sides of a triangle which is
(a) acute angled (b) obtuse angled
(c) right angled (d) none of these

7. The coefficient of t^n in polynomial $(t + {}^{2n+1}C_0)(t + {}^{2n+1}C_1)(t + {}^{2n+1}C_2) \ldots (t + {}^{2n+1}C_n)$ is
(a) 2^{n+1} (b) $2^{2n+1} - 1$
(c) 2^{2n} (d) none of these

8. The total number of 5-digit numbers of different digits in which the digit in the middle is the largest is
(a) $\sum_{n=4}^{9} {}^nP_4$ (b) 33(3!)
(c) 30(3!) (d) none of these

9. If α, β, γ are the roots of $px^3 + qx^2 + r = 0$ then the value of $\begin{vmatrix} 1/\alpha & 1/\beta & 1/\gamma \\ 1/\beta & 1/\gamma & 1/\alpha \\ 1/\gamma & 1/\alpha & 1/\beta \end{vmatrix}$ is
(a) p (b) q (c) 0 (d) r

10. If $\tan A = \dfrac{x^2 - x}{x^2 - x + 1}$ and $\tan B = \dfrac{1}{2x^2 - 2x + 1}$, $0 < A, B < \dfrac{\pi}{2}$, then $A + B =$
(a) $\pi/4$ (b) $\pi/2$ (c) $\pi/3$ (d) $3\pi/4$

11. If A, B, C, D are the smallest positive angles in ascending order of magnitude which have their sines equal to the positive quantity k, then the value of $4\sin\dfrac{A}{2} + 3\sin\dfrac{B}{2} + 2\sin\dfrac{C}{2} + \sin\dfrac{D}{2}$ is equal to
(a) $2\sqrt{1-k}$ (b) $2\sqrt{1+k}$
(c) $2\sqrt{k}$ (d) none of these

12. The value of 'a' for which $at^2 + \sin^{-1}(t^2 - 2t + 2) + \cos^{-1}(t^2 - 2t + 2) = 0$ has a real solution, is
(a) $\pi/2$ (b) $-\pi/2$ (c) $2/\pi$ (d) $-2/\pi$

13. If the straight lines $2x + 3y - 1 = 0$, $x + 2y - 1 = 0$ and $ax + by - 1 = 0$ form a triangle with origin as orthocentre, then (a, b) is given by
(a) $(6, 4)$ (b) $(-3, 3)$
(c) $(-8, 8)$ (d) $(0, 7)$

14. The range of values of 't' such that the angle θ between the pair of tangents drawn from $(t, 0)$ to the circle $x^2 + y^2 = 1$ lies in $\left(\dfrac{\pi}{3}, \pi\right)$, is

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(a) $(-2, -1) \cup (1, 2)$

(b) $\left(-\sqrt{2}, 0\right) \cup \left(0, \sqrt{2}\right)$

(c) $\left(-\sqrt{3}, -\sqrt{2}\right)$

(d) $\left(-\sqrt{3}, -\sqrt{2}\right) \cup \left(\sqrt{2}, \sqrt{3}\right)$

15. The difference between the radii of the largest and the smallest circles which have their centres on the circumference of the circle $x^2 + y^2 - 2x - 4y - 4 = 0$ and pass through the point (α, β) lying outside the given circle, is

(a) 6 (b) $\sqrt{(\alpha+1)^2 + (\beta+2)^2}$

(c) 3 (d) $\sqrt{(\alpha+1)^2 + (\beta+2)^2} - 3$

16. The mirror image of the parabola $y^2 = 4x$ in the tangent to the parabola at the point (1, 2) is

(a) $(x - 1)^2 = 4(y + 1)$

(b) $(x + 1)^2 = 4(y + 1)$

(c) $(x + 1)^2 = 4(y - 1)$

(d) $(x - 1)^2 = 4(y - 1)$

17. If $f\left(2x + \frac{y}{8}, 2x - \frac{y}{8}\right) = xy$, then the value of $f(b, a) + f(a, b)$ is

(a) $a + b$ (b) $a - b$

(c) $a^2 + b^2$ (d) none of these

18. Let a, b and c be the roots of $f(x) = x^3 + x^2 - 5x - 1 = 0$. Then the value of $[a] + [b] + [c]$, where $[\cdot]$ denotes the greatest integer function, is equal to

(a) 1 (b) -2 (c) -4 (d) -3

19. Let $f(x) = x^3 + 3x^2 - 33x - 33$ for $x > 0$ and 'g' be its inverse, then the value of 'k' such that $kg'(2) = 1$ is

(a) 30 (b) -42

(c) 12 (d) none of these

20. The equation of the tangent to the curve $y = \int_{x^2}^{x^3} \frac{dt}{\sqrt{1+t^2}}$ at $x = 1$ is

(a) $\sqrt{2}y + 1 = x$ (b) $\sqrt{3}x + 1 = y$

(c) $\sqrt{3}x + 1 + \sqrt{3} = y$ (d) none of these

21. If $f''(x) > 0$, $\forall\ x \in R$, $f'(3) = 0$ and $g(x) = f(\tan^2 x - 2\tan x + 4)$, $0 < x < \pi/2$, then $g(x)$ is increasing in

(a) $\left(0, \frac{\pi}{4}\right)$ (b) $\left(\frac{\pi}{6}, \frac{\pi}{3}\right)$

(c) $\left(0, \frac{\pi}{3}\right)$ (d) $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

22. The maximum value of $f(t) = t^3 - 3t$ subject to $t^4 + 36 \le 13t^2$, is

(a) 15 (b) 18 (c) 25 (d) ∞

23. The value of $\int_0^{\pi/4} (\tan^9 x + \tan^{11} x) \frac{d}{dx}(x - [x] - [x]^2 - [x]^3)$, where $[x]$ is greatest integral function, is

(a) 1/10 (b) 1/12

(c) 1/13 (d) none of these

24. If $f(t) = a + bt + ct^2$, where $c > 0$ and $b^2 - 4ac < 0$, then the area enclosed by the co-ordinate axes, the line $x = 2$ and the curve $y = f(x)$ is given by

(a) $\frac{1}{3}[4f(1) + f(2)]$

(b) $\frac{1}{2}[f(0) + 4f(1)]$

(c) $\frac{1}{2}[f(0) + 4f(1) + f(2)]$

(d) $\frac{1}{3}[f(0) + 4f(1) + f(2)]$

25. Consider the lines $L_1 : \frac{x+1}{3} = \frac{y+2}{1} = \frac{z+1}{2}$, $L_2 : \frac{x-2}{1} = \frac{y+2}{2} = \frac{z-3}{3}$ Then unit vector perpendicular to both L_1 and L_2 is

(a) $\frac{-\hat{i}+7\hat{j}+7\hat{k}}{\sqrt{99}}$ (b) $\frac{-\hat{i}-7\hat{j}+5\hat{k}}{5\sqrt{3}}$

(c) $\frac{-\hat{i}+7\hat{j}+5\hat{k}}{5\sqrt{3}}$ (d) $\frac{7\hat{i}-7\hat{j}-7\hat{k}}{\sqrt{99}}$

26. If a plane passes through the point (1, 1, 1) and is perpendicular to the line $\frac{x-1}{3} = \frac{y-1}{0} = \frac{z-1}{4}$, then its perpendicular distance from the origin is

(a) 3/4 units (b) 4/3 units

(c) 7/5 units (d) 1 unit

27. In a meeting, 70% of the members favour and 30% oppose a certain proposal. A member is selected at random and we take $X = 0$, if he opposed and $X = 1$, if he is in favour. Then, $E(X)$ and Var (X) respectively are

(a) $\frac{3}{7}, \frac{5}{17}$ (b) $\frac{13}{15}, \frac{2}{15}$

(c) $\frac{7}{10}, \frac{21}{100}$ (d) $\frac{7}{10}, \frac{23}{100}$

28. Statement I : The determinant of a matrix $\begin{bmatrix} 0 & p-q & p-r \\ q-p & 0 & q-r \\ r-p & r-q & 0 \end{bmatrix}$ is zero.

Statement II : The determinant of a skew symmetric matrix of odd order is zero.

(a) Statment I is true, Statement II is true; Statement II is a correct explanation for Statement I.
(b) Statement I is true, Statement II is true; Statement II is not a correct explanation of Statement I.
(c) Statement I is true, Statement II is false.
(d) Statement I is false, Statement II is true.

29. Statement I : $\tan\alpha + 2\tan 2\alpha + 4\tan 4\alpha + 8\tan 8\alpha + 16\cot 16\alpha = \cot\alpha$

Statement II : $\cot\alpha - \tan\alpha = 2\cot 2\alpha$

(a) Statment I is true, Statement II is true; Statement II is a correct explanation for Statement I.
(b) Statement I is true, Statement II is ture; Statement II is not a correct explanation of Statement I.
(c) Statement I is true, Statement II is false.
(d) Statement I is false, Statement II is true.

30. Statement I : The area of triangle formed by the points $A(2000, 2002)$, $B(2001, 2004)$, $C(2002, 2003)$ is same as area formed by $P(0, 0)$, $Q(1, 2)$, $R(2, 1)$.

Statement II : The area of triangle is constant with respect to transition of coordinate axes.

(a) Statment I is true, Statement II is true; Statement II is a correct explanation for Statement I.
(b) Statement I is true, Statement II is ture; Statement II is not a correct explanation of Statement I.
(c) Statement I is true, Statement II is false.
(d) Statement I is false, Statement II is true.

SOLUTIONS

1. (a) : Given $a(l+m)^2 + 2blm + c = 0$
and $a(l+n)^2 + 2bln + c = 0$
$\therefore$ m and n satisfy the equation $a(l+x)^2 + 2blx + c = 0$
$\therefore$ m and n are the roots of $a(l+x)^2 + 2blx + c = 0$
i.e. m and n are the roots of $ax^2 + 2(al+bl)x + c + al^2 = 0$

$\therefore$ mn = product of roots $= \dfrac{c+al^2}{a} = l^2 + \dfrac{c}{a}$

2. (c): $x = 1$ is a root of the equation.
Let a be the first term and d be the common difference of given A.P.

$t_p = a + (p-1)d = \dfrac{1}{q}$ ...(1)

and $t_q = a + (q-1)d = \dfrac{1}{p}$ ...(2)

Solving (1) and (2), $a = d = \dfrac{1}{pq}$

$\therefore$ $t_{pq} = a + (pq-1)d = 1$

$\therefore$ t_{pq} is the root of the given equation.

3. (b) : Since 2^y is positive for all values of y
$\therefore$ $(x-8)(x-10)$ should be positive. Therefore $x > 10$ or $x < 8$.
Since 2^y is a power of 2, both $(x-10)$ and $(x-8)$ should be powers of 2. *i.e.* 2, 4, 8, 16, ...
This is possible when $x = 12$ and $x = 6$. So (12, 3) and (6, 3) are the only solutions.
$\therefore$ Number of ordered pair (x, y) is 2.

4. (a) : $\sin^2\alpha + \sin\alpha\cos\alpha = [t]$

$\dfrac{1-\cos 2\alpha}{2} + \dfrac{\sin 2\alpha}{2} = [t]$

$\Rightarrow$ $\sin 2\alpha - \cos 2\alpha = 2[t] - 1$

$\sqrt{2}\{\sin 2\alpha\cos\pi/4 - \cos 2\alpha\sin\pi/4\} = 2[t]-1$

$\Rightarrow$ $\sqrt{2}\{\sin(2\alpha - \pi/4)\} = 2[t]-1$

$\Rightarrow$ $-\sqrt{2} \le \{2[t]-1\} \le \sqrt{2}$

$\Rightarrow$ $\dfrac{1-\sqrt{2}}{2} \le [t] \le \dfrac{1+\sqrt{2}}{2} \Rightarrow [t] = 0, 1$

$\Rightarrow$ $t \in [0, 1) \cup [1, 2)$

5. (c): Given, $|z| = \dfrac{1}{2}$ ...(1)

Let $\omega = -1 + 4z \Rightarrow z = \dfrac{\omega+1}{4}$

From (1), $\dfrac{|\omega+1|}{4} = \dfrac{1}{2} \Rightarrow |\omega+1| = 2$

$\therefore$ ω lies on a circle of radius 2.

6. (b) : Given $b^2 = ac$ ($\because$ a, b, c are in G.P.)
and $2(\ln 2b - \ln 3c) = \ln a - \ln 2b + \ln 3c - \ln a$
[$\because$ Given terms are in A.P.]

$\Rightarrow$ $\ln\left(\dfrac{2b}{3c}\right)^2 = \ln\left(\dfrac{3c}{2b}\right) \Rightarrow b = \dfrac{3c}{2}$

Now, $a = \dfrac{b^2}{c} = \dfrac{3b}{2} = \dfrac{9c}{4}$

$\therefore$ a is the largest side.

$$\text{Now, } \cos A = \frac{b^2+c^2-a^2}{2bc} = \frac{\frac{9c^2}{4}+c^2-\frac{81}{16}c^2}{2\times\frac{3c}{2}\times c} = \text{negative}$$

$\therefore$ $A > 90°$ $\therefore$ Triangle is obtuse.

7. (c): $(t + {}^{2n+1}C_0)(t + {}^{2n+1}C_1)(t + {}^{2n+1}C_2)......(t + {}^{2n+1}C_n)$

$= t^{n+1} + t^n({}^{2n+1}C_0 + {}^{2n+1}C_1 + ... + {}^{2n+1}C_n) + ...$

$\therefore$ Coefficient of t^n (say) P

$= {}^{2n+1}C_0 + {}^{2n+1}C_1 + {}^{2n+1}C_2 + ... + {}^{2n+1}C_n$

$\Rightarrow P = {}^{2n+1}C_{2n+1} + {}^{2n+1}C_{2n} + {}^{2n+1}C_{2n-1} + ... + {}^{2n+1}C_{n+1}$

$\therefore$ $2P = 2^{2n+1}$ (on adding) $\quad (\because {}^nC_r = {}^nC_{n-r})$

$\therefore$ $P = 2^{2n}$

8. (d) : The smallest number, which can occur in the middle is 4.

The number of numbers with 4 in the middle

$= {}^4P_4 - {}^3P_3$

($\therefore$ The other four places are to be filled by 0, 1, 2 and 3, and a number cannot begin with 0)

Similarly, the number of numbers with 5 in the middle $= {}^5P_4 - {}^4P_3$, etc.

$\therefore$ The required number of numbers

$= ({}^4P_4 - {}^3P_3) + ({}^5P_4 - {}^4P_3) + ({}^6P_4 - {}^5P_3) + ... + ({}^9P_4 - {}^8P_3)$

$$= \sum_{n=4}^{9} ({}^nP_4 - {}^{n-1}P_3)$$

9. (c): Let the value of the determinant be Δ.

Now $C_1 \to C_1 + C_2 + C_3$ gives

$$\Delta = \frac{(\alpha\beta+\beta\gamma+\gamma\alpha)}{\alpha\beta\gamma}\begin{vmatrix} 1 & 1/\beta & 1/\gamma \\ 1 & 1/\gamma & 1/\alpha \\ 1 & 1/\alpha & 1/\beta \end{vmatrix}$$

From the given equation, $\alpha\beta + \beta\gamma + \gamma\alpha = 0$.

So, $\Delta = 0$

10. (a) : $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$

$$= \frac{\frac{a}{a+1}+\frac{1}{2a+1}}{1-\frac{a}{(a+1)(2a+1)}} \text{ where } x^2 - x = a$$

$$= \frac{\frac{a}{a+1}+\frac{1}{2a+1}}{1-\left\{\frac{1}{a+1}-\frac{1}{2a+1}\right\}} = \frac{\frac{a}{a+1}+\frac{1}{2a+1}}{\frac{a}{a+1}+\frac{1}{2a+1}} = 1$$

$$\text{Hence } A+B = \frac{\pi}{4} \quad \left\{\text{Note: } A+B \text{ can not be } \frac{5\pi}{4}\right\}$$

11. (b) : $A < B < C < D$ and $\sin A = \sin B = \sin C = \sin D = k$

$\Rightarrow$ $B = \pi - A,\ C = 2\pi + A,\ D = 3\pi - A$

So, the given expression is equal to

$$4\sin\frac{A}{2} + 3\sin\left(\frac{\pi - A}{2}\right) + 2\sin\frac{2\pi + A}{2} + \sin\frac{3\pi - A}{2}$$

$$= 4\sin\frac{A}{2} + 3\cos\frac{A}{2} - 2\sin\frac{A}{2} - \cos\frac{A}{2}$$

$$= 2\left(\sin\frac{A}{2} + \cos\frac{A}{2}\right) = 2\sqrt{1 + 2\sin\frac{A}{2}\cos\frac{A}{2}}$$

$$= 2\sqrt{1+k}$$

12. (b) : The given equation is

$at^2 + \sin^{-1}\{(t-1)^2 + 1\} + \cos^{-1}\{(t-1)^2 + 1\} = 0$

$\because$ $-1 \le (t-1)^2 + 1 \le 1 \Rightarrow t = 1$

So, we have $a + \dfrac{\pi}{2} = 0 \Rightarrow a = -\dfrac{\pi}{2}$

13. (c): Equation of AO is $2x + 3y - 1 + \lambda(x + 2y - 1) = 0$

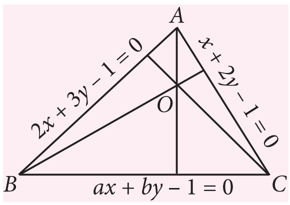
Where $\lambda = -1$, since the line passes through the origin.

So, $x + y = 0$

Since AO is perpendicular to BC, $(-1)\left(-\dfrac{a}{b}\right) = -1$

$\therefore$ $a = -b$

Similarly, $(2x + 3y - 1) + \mu(ax - ay - 1) = 0$ will be the equation of BO for $\mu = -1$.



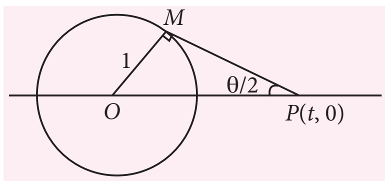
BO is perpendicular to AC

$$\Rightarrow \left\{-\frac{(2-a)}{3+a}\right\}\left(-\frac{1}{2}\right) = -1$$

$\therefore$ $a = -8,\ b = 8$.

14. (a) : We have $\dfrac{\pi}{3} < \theta < \pi$

$$i.e. \ \frac{\pi}{6} < \frac{\theta}{2} < \frac{\pi}{2} \ \ i.e. \ \frac{1}{2} < \sin\left(\frac{\theta}{2}\right) < 1$$



i.e. $\frac{1}{2} < \frac{1}{t} < 1 \qquad \left[\because \sin\left(\frac{\theta}{2}\right) = \frac{1}{t} \text{(See fig.)}\right]$

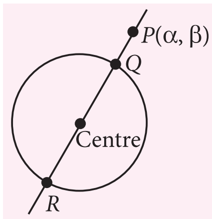
i.e. $1 < t < 2$

There can be symmetrical points on the –ve x-axis too. Hence, we have $t \in (-2, -1) \cup (1, 2)$.

15. (a) : The given circle is $(x-1)^2 + (y-2)^2 = 9$ has radius = 3

The points on the circle which are nearest and farthest to the point $P(\alpha, \beta)$ are Q and R respectively (see fig.)

Thus, the circle centred at Q having radius PQ will be the smallest required circle while the circle centred at R having radius PR will be the largest required circle. Hence, difference between their radii $= PR - PQ = QR = 6$.



16. (c): Any point on the given parabola is $(t^2, 2t)$. The equation of the tangent at (1, 2) is $x - y + 1 = 0$

The image (h, k) of the point $(t^2, 2t)$ in $x - y + 1 = 0$ is given by

$$\frac{h-t^2}{1} = \frac{k-2t}{-1} = -\frac{2(t^2-2t+1)}{1+1}$$

$\therefore\ h = t^2 - t^2 + 2t - 1 = 2t - 1$ and

$k = 2t + t^2 - 2t + 1 = t^2 + 1$

Eliminating t from $h = 2t - 1$ and $k = t^2 + 1$,

We get $(h+1)^2 = 4(k-1)$

17. (d) : Let $2x + \frac{y}{8} = \alpha$ and $2x - \frac{y}{8} = \beta$,

Then $x = \frac{\alpha+\beta}{4}$ and $y = 4(\alpha - \beta)$

Given, $f\left(2x + \frac{y}{8}, 2x - \frac{y}{8}\right) = xy$

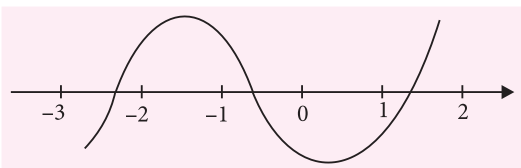
$\Rightarrow f(\alpha,\beta) = \alpha^2 - \beta^2$

$\Rightarrow f(a, b) + f(b, a) = a^2 - b^2 + b^2 - a^2 = 0$

18. (d) : Let $a < b < c$.

The values of $f(0) = -1, f(1) = -4, f(2) = 1, f(-1) = 4, f(-2) = 5, f(-3) = -4$

So graph of $f(x)$ is as shown in the figure



Hence $a \in (-3, -2)$, $b \in (-1, 0)$ and $c \in (1, 2)$.Clearly

$[a] + [b] + [c] = -3 - 1 + 1 = -3$

19. (d) : $g(f(x)) = x \Rightarrow g'(f(x)) \,.\, f'(x) = 1$

$$\Rightarrow\ g'(f(x)) = \frac{1}{f'(x)}$$

Now, $\ f(x) = 2 \Rightarrow x^3 + 3x^2 - 33x - 33 = 2$

$\Rightarrow x^3 + 3x^2 - 33x - 35 = 0$

$\Rightarrow x^3 - 5x^2 + 8x^2 - 40x + 7x - 35 = 0$

or $(x-5)(x^2 + 8x + 7) = 0$

$\Rightarrow (x-5)(x+1)(x+7) = 0$

$\therefore\ x = -7, -1, 5$. Thus we have

$k = f'(-1) = 3(-1)^2 + 6(-1) - 33 = 3 - 6 - 33 = -36$

$k = f'(-7) = 3(-7)^2 + 6(-7) - 33 = 147 - 42 - 33 = 72$

$k = f'(5) = 3(5)^2 + 6(5) - 33 = 75 + 30 - 33 = 72$

20. (a) : Differentiating the given equation, we get

$$\frac{dy}{dx} = \left[\frac{1}{\sqrt{1+t^2}}\right]_{t=x^3} \frac{d}{dx}(x^3) - \left[\frac{1}{\sqrt{1+t^2}}\right]_{t=x^2} \frac{d}{dx}(x^2)$$

$$= \frac{3x^2}{\sqrt{1+x^6}} - \frac{2x}{\sqrt{1+x^4}}$$

Therefore the slope of the required tangent is

$$\left(\frac{dy}{dx}\right)_{x=1} = \frac{3}{\sqrt{2}} - \frac{2}{\sqrt{2}} = \frac{1}{\sqrt{2}}$$

This tangent passes through the point $x = 1$

$$\therefore\ y(1) = \int_1^1 \frac{dt}{\sqrt{1+t^2}} = 0$$

So, required equation is $y - 0 = \frac{1}{\sqrt{2}}(x-1)$

$\Rightarrow\ \sqrt{2}y + 1 = x$

21. (d) : $g'(x) = (f'((\tan x - 1)^2 + 3))2(\tan x - 1)\sec^2 x$.

Since $f''(x) > 0 \Rightarrow f'(x)$ is increasing

So, $f'((\tan x - 1)^2 + 3) > f'(3) = 0$

$$\forall\ x \in \left(0, \frac{\pi}{4}\right) \cup \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$$

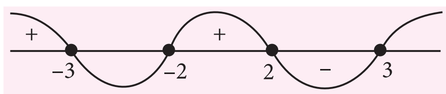
For $g(x)$ to be increasing $(\tan x - 1) > 0$ for $x \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

So, $g(x)$ is increasing in $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$.

22. (b) : $t^4 + 36 \le 13t^2 \Rightarrow t^4 - 13t^2 + 36 \le 0$

$\Rightarrow (t-3)(t+3)(t+2)(t-2) \le 0$

Using wavy curve method, $t \in [-3, -2] \cup [2, 3]$



Now, $f(t) = t^3 - 3t$

$\Rightarrow f'(t) = 3t^2 - 3 > 0$, if $t \in [-3, -2] \cup [2, 3]$

$\therefore f(t)$ increases as $t \in [-3, -2] \cup [2, 3]$

$\therefore$ Maximum value of $f(t)$ = Maximum of $\{f(-3), f(-2), f(2), f(3)\}$

= Maximum of $\{-18, -2, 2, 18\} = 18$

23. (a) : Since, $[x] = 0$ in $\left[0, \frac{\pi}{4}\right]$

We have, $I = \int_0^{\pi/4} (\tan^2 x + 1)\tan^9 x dx$

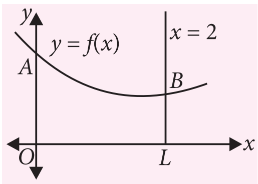
$= \int_0^{\pi/4} \sec^2 x \cdot \tan^9 x \, dx$

Put, $\tan x = t \Rightarrow \sec^2 x \, dx = dt$.

$\therefore \quad I = \int_0^1 t^9 dt = \left[\frac{t^{10}}{10}\right]_0^1 = \frac{1}{10}$

24. (d) : Area of $OABL = \int_0^2 y dx = \int_0^2 (a + bx + cx^2) dx$

$= \left(2a + 2b + \frac{8}{3}c\right) = \frac{1}{3}[6a + 6b + 8c]$



But, $f(x) = a + bx + cx^2$

$f(0) = a$, $f(1) = a + b + c$

$f(2) = a + 2b + 4c$

Also, $\frac{1}{3}\{f(0) + 4f(1) + f(2)\}$

$= \frac{1}{3}\{a + 4(a + b + c) + (a + 2b + 4c)\} = \frac{1}{3}\{6a + 6b + 8c\}$

25. (b) : The direction ratios of L_1 and L_2 are (3, 1, 2) and (1, 2, 3) and so the vector perpendicular to both is

given by $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & 2 \\ 1 & 2 & 3 \end{vmatrix} = -\hat{i} - 7\hat{j} + 5\hat{k}$

Then the unit vector is $\dfrac{-\hat{i} - 7\hat{j} + 5\hat{k}}{5\sqrt{3}}$

26. (c): Equation of plane passing through (1, 1, 1) and perpendicular to line $\frac{x-1}{3} = \frac{y-1}{0} = \frac{z-1}{4}$ is

$3(x - 1) + 0(y - 1) + 4(z - 1) = 0$

$\Rightarrow \quad 3x + 4z - 7 = 0$

Distance of plane from the origin $= \left|\frac{0 + 0 - 7}{\sqrt{9 + 16}}\right| = \frac{7}{5}$ units

27. (c): It is given that, $P(X = 0) = 30\% = \frac{30}{100} = \frac{3}{10}$

$P(X = 1) = 70\% = \frac{70}{100} = \frac{7}{10}$

Therefore, the probability distribution is as follows

X	0	1
$P(X)$	$\frac{3}{10}$	$\frac{7}{10}$

$\therefore \quad$ Mean of $X = E(X) = \Sigma X P(X)$

$= 0 \times \frac{3}{10} + 1 \times \frac{7}{10} = \frac{7}{10}$

Variance of $X = \Sigma X^2 P(X) - (\text{Mean})^2$

$= (0)^2 \times \frac{3}{10} + (1)^2 \times \frac{7}{10} - \left(\frac{7}{10}\right)^2 = \frac{7}{10} - \frac{49}{100} = \frac{21}{100}$

28. (a) : The determinant of a skew symmetric matrix of odd order is zero

i.e., $|A^T| = |-A| \Rightarrow |A| = -|A|$

$\Rightarrow \quad 2|A| = 0 \Rightarrow |A| = 0$

29. (a) : $\cot\alpha - \tan\alpha = 2\cot 2\alpha$

$\Rightarrow \quad \tan\alpha + 2\tan 2\alpha + 4\tan 4\alpha + 8\tan 8\alpha + 16\cot 16\alpha = \cot\alpha$

30 (a) : Area of triangle is unaltered by shifting the origin to any point.

If origin is shifted to (2000, 2002), A, B, C become $P(0, 0)$ $Q(1, 2)$, $R(2, 1)$

Both are true.

EXAM DATES 2016

JEE Main	:	3rd April (offline), 9th & 10th April (online)
VITEEE	:	6th to 17th April
MGIMS	:	17th April
AMU (Engg.)	:	24th April
Kerala PET	:	25th & 26th April
Kerala PMT	:	27th & 28th April
APEAMCET (Engg. & Med.)	:	29th April
AIPMT	:	1st May
COMED K	:	8th May
Karnataka CET	:	4th & 5th May
BITSAT	:	14th to 28th May
WB JEE	:	17th May
JEE Advanced	:	22nd May
AIIMS	:	29th May
AMU (Med.)	:	1st June
JIPMER	:	5th June

QUANTITATIVE APTITUDE TEST

Useful for Bank PO, Specialist Officers & Clerical Cadre, BCA, MAT, CSAT, CDS and other such examinations.

1. A young girl counted in the following way on the fingers of her left hand. She started calling the thumb 1, the index finger 2, middle finger 3, ring finger 4, little finger 5, then reversed direction, calling the ring finger 6, middle finger 7, index finger 8, thumb 9 and then back to the index finger for 10, middle finger for 11, and so on. She counted up to 1994. She ended on her
   (a) Thumb (b) Index finger
   (c) Middle finger (d) Ring finger

2. $A, B, C, D, \ldots, X, Y, Z$ are the players who participated in a tournament. Everyone played with every other player exactly once. A win scores 2 points, a draw scores 1 point and a lose scores 0 point. None of the matches ended in a draw. No two players scored the same score. At the end of the tournament, a ranking list is published which is in accordance with the alphabetical order. Then :
   (a) M wins over N (b) N wins over M
   (c) E wins over D (d) None of these

3. The following table presents the sweetness of different items relative to sucrose, whose sweetness is taken to be 1.00.

Lactose	0.16
Maltose	0.32
Glucose	0.74
Sucrose	1.00
Fructose	1.70
Saccharin	675.00

   Approximately how many times sweeter than sucrose is a mixture consisting of glucose; sucrose and fructose in the ratio of 1 : 2 : 3?
   (a) 2.3 (b) 1.0
   (c) 0.3 (d) None of these

4. Anjali bought some chocolates from Nestle's exclusive shop, she gave to Amit one less than half of what she had initially. Then she had given 3 chocolates to Bablu and then half of the chocolates which she had then gave to Charles. Thus finally she gave one chocolate to Deepak and the remaining one she ate herself. The number of chocolates she had purchased.
   (a) 9 (b) 12 (c) 10 (d) 15

5. A water tank has three taps : A, B and C. A fills 4 buckets in 24 mins, B fills 8 buckets in 1 hr and C fills 2 buckets in 20 mins. If all the taps are opened together, a full tank is emptied in 2 hrs. If a bucket contains 5 L water, what is the capacity of the tank?
   (a) 120 L (b) 240 L (c) 180 L (d) 60 L

6. The number of votes not cast for the Praja Party increased by 25% in the National General Election over those not cast for it in the previous Assembly Polls, and the Praja Party lost by a majority twice as large as that by which it had won the Assembly Polls. If a total 2,60,000 people voted each time, how many voted for the Praja Party in the previous Assembly Polls?
   (a) 1,10,000 (b) 1,50,000
   (c) 1,40,000 (d) 1,20,000

7. A survey on a sample of 25 new cars being sold at a local auto dealer was conducted to see which of the three popular options-air conditioning, radio and power windows-were already installed. The survey found : 15 had air conditioning, 2 had air conditioning and power windows but no radios, 12 had radio, 6 had air conditioning and radio but no power windows, 11 had power windows, 4 had radio and power windows, 3 had all three options. What is the number of cars that had none of the options?
   (a) 4 (b) 3 (c) 1 (d) 2

8. A farmer has decided to build a wire fence along one straight side of his property. For this, he planned to place several fence-posts at 6 m intervals, with posts fixed at both ends of the side. After he bought the posts and wire, he found that the number of posts he had bought was 5 less than required. However, he discovered that the number of posts he had bought would be just sufficient if he spaced them 8 m apart. What is the length of the side of his property and how many posts did he buy?

(a) 100 m, 15 (b) 100 m, 16
(c) 120 m, 15 (d) 120 m, 16

9. Consider two different cloth-cutting processes. In the first one, n circular cloth pieces are cut from a square cloth piece of side a in the following steps : the original square of side a is divided into n smaller squares, not necessarily of the same size; then a circle of maximum possible area is cut from each of the smaller squares. In the second process, only one circle of maximum possible area is cut from the square of side a and the process ends there. The cloth pieces remaining after cutting the circles are scrapped in both the processes. The ratio of the total area of scrap cloth generated in the former to that in the latter is

(a) 1 : 1 (b) $\sqrt{2}:1$
(c) $\dfrac{n(4-\pi)}{4n-\pi}$ (d) $\dfrac{4n-\pi}{n(4-\pi)}$

10. Three bells chime at intervals of 18 mins, 24 mins and 32 mins respectively. At a certain time, they begin to chime together. What length of time will elapse before they chime together again?

(a) 2 hrs and 24 mins (b) 4 hrs and 48 mins
(c) 1 hrs and 36 mins (d) 5 hrs

11. I started climbing up the hill at 6 a.m. and reached the top of the temple at 6 p.m. Next day, I started coming down at 6 a.m. and reached the foothill at 6 p.m. I walked on the same road. The road is so short that only one person can walk on it. Although given I varied my pace on my way, I never stopped on my way. Then which of the following must be true?

(a) My average speed downhill was greater than that of uphill
(b) At noon, I was at the same spot on both the days.
(c) My average speed uphill was greater than downhill.
(d) None of these

12. A rectangle $PRSU$, is divided into two smaller rectangles $PQTU$, and $QRST$ by the line TQ.PQ = 10 cm. QR = 5 cm and RS = 10 cm. Points A, B, F are within rectangle $PQTU$, and points C, D, E are within the rectangle $QRST$. The closest pair of points among the pairs (A, C), (A, D), (A, E), (F, C), (F, D), (F, E), (B, C), (B, D), (B, E) are $10\sqrt{3}$ cm apart.

Which of the following statements is necessarily true?

(a) The closest pair of points among the six given points cannot be (F, C).
(b) Distance between A and B is greater than that between F and C.
(c) The closest pair of points among the six given points is (C, D), (D, E) or (C, E).
(d) Cannot be determined

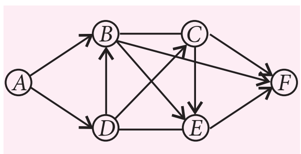
13. A watch dealer incurs an expense of ₹ 150 for producing every watch. He also incurs an additional expenditure of ₹ 30,000, which is independent of the number of watch produced. If he is able to sell a watch during the season, he sells it for ₹ 250. If he fails to do so he has to sell each watch for ₹ 100.
If he is able to sell only 1200 out of 1500 watches he has made in the season, then he has made a profit of :

(a) ₹ 90,000 (b) ₹ 75,000
(c) ₹ 45,000 (d) ₹ 60,000

14. A child was asked to add first few natural numbers (that is, 1 + 2 + 3 + ...) so long his patience permitted. As he stopped he gave the sum as 575. When the teacher declared the result wrong the child discovered he had missed one number in the sequence during addition. The number he missed was

(a) Less than 10 (b) 10
(c) 15 (d) More than 15

15. The given figure shows the network connecting cities A, B, C, D, E and F. The arrows indicate permissible direction of travel. What is the number of distinct path from A to F?



(a) 9 (b) 10
(c) 11 (d) None of these

16. An intelligence agency forms a code of two distinct digits selected from 0, 1, 2, ..., 9 such that the first digit of the code is nonzero. The code, handwritten on a slip, can however potentially create confusion, when read upside down-for example, the code 91 may appear as 16. How many codes are there for which no such confusion can arise?

(a) 80 (b) 78
(c) 71 (d) 69

17. Read the information given below and answer the question that follows:

The batting average (BA) of a test batsman is computed from runs scored and innings played-completed innings and incomplete innings (not out) in the following manner

r_1 = number of runs scored in completed innings
n_1 = number of completed innings
r_2 = number of runs scored in incomplete innings
n_2 = number of incomplete innings

$$BA = \frac{r_1 + r_2}{n_1}$$

To better assess a batsman's accomplishments, the ICC is considering two other measures $MB\ A_1$ and $MB\ A_2$ defined as follows :

$$MB\ A_1 = \frac{r_1}{n_1} + \frac{r_2}{n_2}\max\left[0, \left(\frac{r_1}{n_2} - \frac{r_1}{n_1}\right)\right] : MB\ A_2 = \frac{r_1 + r_2}{n_1 + n_2}$$

An experienced cricketer with no incomplete innings has a BA of 50. The next time he bats, the innings is incomplete and he scores 45 runs. It can be inferred that

(a) BA and $MB\ A_1$ both will decrease.
(b) BA will increase and $MB\ A_2$ will decrease.
(c) BA will increase and not enough data is available to assess change in $MB\ A_1$ and $MB\ A_2$.
(d) None of these

18. An airline has a certain free luggage allowance and charges for excess luggage at a fixed rate per kg. Two passengers, Ananya and Krishi have 60 kg of luggage between them, and are charged ₹ 1200 and ₹ 2400, respectively for excess luggage. Had the entire luggage belonged to one of them, the excess luggage charge would have been ₹ 5400.
What is the weight of Krishi's luggage?

(a) 20 kg (b) 25 kg
(c) 30 kg (d) 35 kg

Directions (19-20) : Answer the questions based upon following information :
Each of the question is followed by two statements, I and II.

Choose 1. If the question can be answered with the help of statement I alone.

Choose 2. If the question can be answered with the help of statement II alone.

Choose 3. If the question can be answered with the help of both statements together.

Choose 4. If the question cannot be answered even with the help of both statements together.

19. After what time will the two persons Tez and Gati meet while moving around the circular track? Both of them start at the same point and at the same time.

I. Tez moves at a constant speed of 5 m/s, while Gati starts at a speed of 2 m/s and increased his speed by 0.5 m/s at the end of every second thereafter.
II. Gati can complete one entire lap in exactly 10 s.

(a) 1 (b) 2 (c) 3 (d) 4

20. What is the area bounded by the two lines and the coordinate axes in the first quadrant?

I. The lines intersect at a point which also lies on the lines $3x - 4y = 1$ and $7x - 8y = 5$.
II. The lines are perpendicular, and one of them intersects the Y-axis at an intercept of 4.

(a) 1 (b) 2 (c) 3 (d) 4

21. Mr. Bankatlal acted as a judge for the beauty contest. There were four participants, viz. Ms. Andhra Pradesh, Ms. Uttar Pradesh, Ms. West Bengal and Ms. Maharashtra, Mrs. Bankatlal, who was very anxious about the result asked him about it as soon as he was back home. Mr. Bankatlal just told that the one who was wearing the yellow saree won the contest. When Mrs. Bankatlal pressed for further details, he elaborated as follows:

A. All of them were sitting in a row.
B. All of them wore sarees of different colours, *viz.* green, yellow, white, red.
C. There was only one runner-up and she was sitting beside Ms. Maharashtra.
D. The runner-up was wearing the green saree.
E. Ms. West Bengal was not sitting at the ends and was not the runner up.

F. Ms. Maharashtra was wearing white saree.

G. Ms. Andhra Pradesh was not wearing the green saree.

H. Participants wearing yellow saree and white saree were at the ends.

Who wore the red saree?

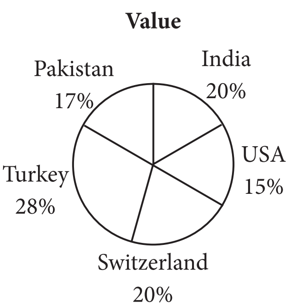
(a) Ms. Andhra Pradesh

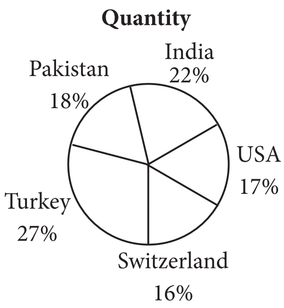
(b) Ms. West Bengal

(c) Ms. Uttar Pradesh

(d) Ms. Maharashtra

Directions (22-23) : The pie charts give the data about a textile manufacturing unit exporting to different countries.





The total value is 5760 million rupees and the total quantity is 1.055 million tonnes.

22. Which country has the highest price for its supply?

(a) Pakistan (b) Turkey

(c) Switzerland (d) India

23. What is the price in ₹ per kg for Turkey?

(a) 6.3 (b) 5.6 (c) 4.8 (d) 4.5

24. A, B and C are defined as follows :

$A = (2.000004) \div [(2.000004)^2 + (4.000008)]$,

$B = (3.000003) \div [(3.000003)^2 + (9.000009)]$,

$C = (4.000002) \div [(4.000002)^2 + (8.000004)]$,

Which of the following is true about the values of the above three expression?

(a) All of them lie between 0.18 and 0.2

(b) A is twice of C

(c) C is the smallest

(d) B is the smallest

25. You can collect rubies and emeralds as many you can. Each ruby is worth ₹ 4 crores and each emerald is wroth of ₹ 5 crore. Each ruby weights 0.3 kg and each emerald weighs 0.4 kg. Your bag can carry at the most 12 kg. What you should collect to get the maximum wealth?

(a) 20 rubies and 15 emeralds

(b) 40 rubies

(c) 28 rubies and 9 emeralds

(d) None of these

ANSWER KEYS

1.	(b)	2.	(a)	3.	(d)	4.	(b)	5.	(b)
6.	(c)	7.	(d)	8.	(d)	9.	(a)	10.	(b)
11.	(d)	12.	(d)	13.	(b)	14.	(d)	15.	(d)
16.	(d)	17.	(b)	18.	(d)	19.	(d)	20.	(c)
21.	(b)	22.	(c)	23.	(b)	24.	(d)	25.	(b)

Math Archives 10 Best Problems

Math Archives, as the title itself suggests, is a collection of various challenging problems related to the topics of IIT-JEE Syllabus. This section is basically aimed at providing an extra insight and knowledge to the candidates preparing for IIT-JEE. In every issue of MT, challenging problems are offered with detailed solution. The readers' comments and suggestions regarding the problems and solutions offered are always welcome.

1. The centre of a square $ABCD$ is at z_0. If A is z_1, then the centroid of the ΔABC is

(a) $2z_0 - i(z_1 - z_0)$ (b) $z_0 + i\left(\dfrac{z_1 - z_0}{3}\right)$

(c) $\dfrac{z_0 + iz_1}{3}$ (d) $\dfrac{2}{3}(z_1 - z_0)$

2. The reflection of the complex number $(2 - i)$ in the straight line $iz = \bar{z}$ is

(a) $4 - 3i$ (b) $3 + 4i$ (c) $2 + i$ (d) $1 - 2i$

3. If sum of the arguments of all the roots of the equation $z^n = k$ (k is real) is odd multiple of π, then it can be true when

(i) n is odd and $k > 0$
(ii) n is odd and $k < 0$
(iii) n is even and $k > 0$
(iv) n is even and $k < 0$

(a) (i) and (iii) are correct
(b) (i) and (iv) are correct
(c) (ii) and (iii) are correct
(d) (ii) and (iv) are correct

4. Let $f : R \to \left(0, \dfrac{\pi}{2}\right]$ be a function defined by $f(x) = \cot^{-1}(x^2 + 4x + \alpha^2 - \alpha)$, then complete set of values of α for which $f(x)$ is onto, is

(a) $\left[\dfrac{1-\sqrt{17}}{2}, \dfrac{1+\sqrt{17}}{2}\right]$ (b) $\dfrac{1 \pm \sqrt{17}}{2}$

(c) $\left(-\infty, \dfrac{1-\sqrt{17}}{2}\right] \cup \left[\dfrac{1+\sqrt{17}}{2}, \infty\right)$

(d) none of these

5. Let $g(x)$ and $f(x)$ be twice differentiable functions in R and $f(2) = 8$, $g(2) = 0$, $f(4) = 10$ and $g(4) = 8$, then

(a) $g'(x) > 4f'(x) \forall x \in (2, 4)$
(b) $g(x) > f(x) \forall\, x \in (2, 4)$
(c) $3g'(x) = 4f'(x)$ for at least one $x \in (2, 4)$
(d) $g'(x) = 4f'(x)$ for at least one $x \in (2, 4)$

6. If A, G, H be respectively the A.M., G.M. and the H.M. between two positive numbers and if $xA = yG = zH$ where x, y, z are non zero positive quantities, then x, y, z are in

(a) A.P. (b) G.P. (c) H.P. (d) A.G.P

7. Given $\displaystyle\int_0^{\pi/2} \frac{dx}{1 + \sin x + \cos x} = \ln 2$, then the value of the definite integral $\displaystyle\int_0^{\pi/2} \frac{\sin x}{1 + \sin x + \cos x} dx$ is

(a) $\dfrac{1}{2}\ln 2$ (b) $\dfrac{\pi}{2} - \ln 2$

(c) $\dfrac{\pi}{4} - \dfrac{1}{2}\ln 2$ (d) $\dfrac{\pi}{2} + \ln 2$

8. Let $y = \{x\}^{[x]}$ where $\{x\}$ denotes the fractional part of x and $[x]$ denotes greatest integer $\le x$, then $\displaystyle\int_0^3 y\,dx =$

(a) $\dfrac{11}{6}$ (b) $\dfrac{2}{3}$ (c) 1 (d) $\dfrac{5}{6}$

9. A quadratic equation $f(x) = ax^2 + bx + c = 0$ $(a \ne 0)$ has positive distinct roots reciprocal of each other. Which one is correct?

(a) $f'(1) = 0$ (b) $af'(1) < 0$
(c) $af'(1) > 0$
(d) nothing can be said about $af'(1)$

By : **Prof. Shyam Bhushan,** Director, Narayana IIT Academy, Jamshedpur. Mob. : 09334870021

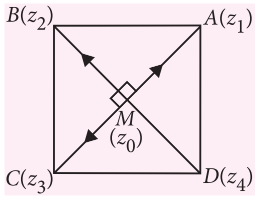
10. Each of 10 passengers board any of the three buses randomly, which had no passenger initially. The probability that each bus has got at least one passenger, is

(a) $\dfrac{{}^{10}P_3\, 3^7}{3^{10}}$ (b) $1-\dfrac{{}^{10}C_3\ 3^7}{3^{10}}$

(c) $1-\dfrac{2^{10}}{3^{10}}$ (d) $\dfrac{3^{10}-3\cdot 2^{10}+3}{3^{10}}$

SOLUTIONS

1. (b) : From the figure, $z_2 - z_0 = i(z_1 - z_0)$



$\Rightarrow\ z_2 = z_0 + i(z_1 - z_0)$ and $z_3 - z_0 = -(z_1 - z_0)$

$\Rightarrow\ z_3 = 2z_0 - z_1$

Hence, centroid of ΔABC

$=\dfrac{z_1+z_2+z_3}{3}=z_0+i\left(\dfrac{z_1-z_0}{3}\right)$

2. (d) : $iz=\overline{z}$

$\Rightarrow\ i(x + iy) = x - iy\ \Rightarrow\ i(x + y) - (x + y) = 0$

$\Rightarrow\ (i - 1)(x + y) = 0$

which represents the line $y = -x$

So, reflection of the point $(2, -1)$ in the line $y = -x$ gives the point $(1, -2)$.

3. (c) : When n is odd, and $k > 0$

Sum of arguments = even multiple of π.

When n is odd and $k < 0$

Sum of arguments = odd multiple of π.

When n is even and $k > 0$

Sum of arguments = odd multiple of π.

When n is even and $k < 0$

Sum of arguments = even multiple of π.

4. (b) : Clearly $x^2 + 4x + \alpha^2 - \alpha \ge 0\ \ \forall\ x \in R$ and must take all values of the interval $[0, \infty)$

$\Rightarrow\ D = 0$

i.e., $16 - 4(\alpha^2 - \alpha) = 0\ \Rightarrow\ \alpha^2 - \alpha = 4$

$\Rightarrow\ \alpha=\dfrac{1\pm\sqrt{17}}{2}.$

5. (d) : Let $h(x) = g(x) - 4f(x)$

Verify the Rolle's theorem in $(2, 4)$

Now, $h(2) = g(2) - 4f(2) = 0 - 4 \times 8 = -32$

$h(4) = g(4) - 4f(4) = 8 - 40 = -32$

$\Rightarrow\ h'(x) = 0$ for at least one $x \in (2, 4)$

$\therefore\ g'(x) = 4f'(x)$ for at least one $x \in (2, 4)$.

6. (b) : Let two numbers be a and b

$x.\dfrac{a+b}{2}=y.\sqrt{ab}=z.\dfrac{2ab}{a+b}\ \Rightarrow\ \dfrac{a+b}{2\sqrt{ab}}=\dfrac{y}{x}=\dfrac{z}{y}$

$\Rightarrow\ y^2 = xz\ \Rightarrow x, y, z$ are in G.P.

7. (c) : $I=\displaystyle\int_0^{\pi/2}\frac{1+\sin x+\cos x-1-\cos x}{1+\sin x+\cos x}dx$

$=\displaystyle\int_0^{\pi/2}dx-\int_0^{\pi/2}\frac{dx}{1+\sin x+\cos x}-\int_0^{\pi/2}\frac{\cos x}{1+\sin x+\cos x}dx$

$I=\dfrac{\pi}{4}-\dfrac{1}{2}\ln 2.$

8. (a) :

$\displaystyle\int_0^3 (x-[x])^{[x]}dx=\int_0^1 dx+\int_1^2 (x-1)dx+\int_2^3 (x-2)^2 dx$

Put $t = x - 1,\ \ z = x - 2$

$=1+\displaystyle\int_0^1 t\,dt+\int_0^1 z^2dz=1+\left[\frac{t^2}{2}\right]_0^1+\left[\frac{z^3}{3}\right]_0^1$

$=1+\dfrac{1}{2}+\dfrac{1}{3}=\dfrac{11}{6}.$

9. (b) : $f(x) = ax^2 + bx + c = 0 (a \ne 0)$

Let α and $\dfrac{1}{\alpha}$ be its roots.

$\therefore\ \ \alpha+\dfrac{1}{\alpha}=-\dfrac{b}{a}>2$

$\Rightarrow\ \left(\dfrac{2a+b}{a}\right)<0\ \Rightarrow\ a(2a+b)<0$

$f'(x) = 2ax + b$

$af'(1) = a(2a + b) < 0$

10. (d) : Total ways in which they can be seated in the buses $= 3^{10}$

Number of cases in which each bus has got at least one passenger

$= 3^{10} - {}^3C_1 \times 1 - {}^3C_2(2^{10} - 2)$

$\therefore$ Required probability

$=\dfrac{3^{10}-{}^3C_1\times 1-{}^3C_2\left(2^{10}-2\right)}{3^{10}}=\dfrac{3^{10}-3\cdot 2^{10}+3}{3^{10}}$

YOU ASK WE ANSWER

Do you have a question that you just can't get answered?

Use the vast expertise of our MTG team to get to the bottom of the question. From the serious to the silly, the controversial to the trivial, the team will tackle the questions, easy and tough.

The best questions and their solutions will be printed in this column each month.

1. Find the general equation of the circle passing through two given points $A(x_1, y_1)$ and $B(x_2, y_2)$.

– Surya, A.P.

Ans. Let $P(h, k)$ be any point on the circle passing through points $A(x_1, y_1)$ and $B(x_2, y_2)$. Since the angle in the same segment of a circle is always same. Therefore, $\angle APB = \theta$ or $\pi - \theta$, where θ is some angle.

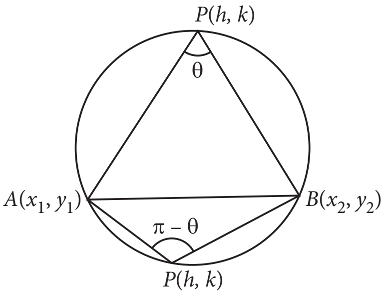
Now, $m_1 = \text{Slope of } AP = \dfrac{k-y_1}{h-x_1}$,

and, $m_2 = \text{Slope of } BP = \dfrac{k-y_2}{h-x_2}$

$$\tan\theta = \pm\frac{m_1-m_2}{1+m_1m_2}$$

$$\Rightarrow \tan\theta = \pm\frac{\dfrac{k-y_1}{h-x_1}-\dfrac{k-y_2}{h-x_2}}{1+\dfrac{k-y_1}{h-x_1}\times\dfrac{k-y_2}{h-x_2}}$$

$$\Rightarrow \tan\theta = \pm\frac{(h-x_2)(k-y_1)-(h-x_1)(k-y_2)}{(h-x_1)(h-x_2)+(k-y_1)(k-y_2)}$$



$\Rightarrow (h-x_1)(h-x_2)+(k-y_1)(k-y_2)$
$= \pm\cot\theta\{(h-x_2)(k-y_1)-(h-x_1)(k-y_2)\}$

Hence, the locus of (h, k) is

$(x-x_1)(x-x_2)+(y-y_1)(y-y_2)$
$= \pm\cot\theta\{(x-x_2)(y-y_1)-(x-x_1)(y-y_2)\}$

$\Rightarrow (x-x_1)(x-x_2)+(y-y_1)(y-y_2)$
$\pm\cot\theta\{x(y_1-y_2)+y(x_2-x_1)+x_1y_2-x_2y_1\}=0$

$$\Rightarrow (x-x_1)(x-x_2)+(y-y_1)(y-y_2) \pm\cot\theta\begin{vmatrix} x & y & 1\\ x_1 & y_1 & 1\\ x_2 & y_2 & 1\end{vmatrix}=0$$

2. Let $f(x)$ be a polynomial of degree three satisfying $f(0) = -1$ and $f(1) = 0$. Also, 0 is a stationary point of $f(x)$, does not have an extremum at $x = 0$, then find the value of the integral $\displaystyle\int\frac{f(x)}{x^3-1}dx$.

– Deepak, W.B.

Ans. Let $f(x) = ax^3 + bx^2 + cx + d$. Then,

$f(0) = -1$ and $f(1) = 0$

$\Rightarrow d = -1$ and $a + b + c + d = 0$

$\Rightarrow d = -1$ and $a + b + c = 1$ …(i)

It is given that $x = 0$ is a stationary point of $f(x)$ but it is not a point of extremum. Therefore,

$f'(0) = 0, f''(0) = 0$ and $f'''(0) \neq 0$

Now, $f(x) = ax^3 + bx^2 + cx + d$

$\Rightarrow f'(x) = 3ax^2 + 2bx + c, f''(x) = 6ax + 2b$ and $f'''(x) = 6a$

$\therefore f'(0) = 0, f''(0) = 0$ and $f'''(0) \neq 0$

$\Rightarrow c = 0, b = 0$ and $a \neq 0$ …(ii)

From equations (i) and (ii), we get

$a = 1, b = c = 0$ and $d = -1$

$\therefore f(x) = x^3 - 1$

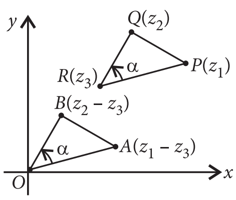
Hence, $\displaystyle\int\frac{f(x)}{x^3-1}dx = \int 1\,dx = x + c$

3. Let z_1, z_2 and z_3 be three complex numbers represented by P, Q and R, respectively. If α is the angle $\angle PRQ$, then prove that

$$\frac{z_2-z_3}{z_1-z_3} = \frac{RQ}{RP}(\cos\alpha + i\sin\alpha)$$

– Neeraj, W.B.

Ans. Let the points A and B represent $z_1 - z_3$ and $z_2 - z_3$, respectively, so that $RP = OA$, $RQ = OB$ and $PQ = AB$. Therefore ΔPQR and ΔABO are congruent and hence $\angle AOB = \alpha$.



$$\alpha = \arg\left(\frac{z_2-z_3}{z_1-z_3}\right)$$

Therefore, $\dfrac{z_2-z_3}{z_1-z_3} = \dfrac{|z_2-z_3|}{|z_1-z_3|}(\cos\alpha + i\sin\alpha)$

$$= \frac{RQ}{RP}(\cos\alpha + i\sin\alpha)$$

AMU PRACTICE PAPER 2016

Aligarh Muslim University

1. The number of solutions of the system of equations $\text{Re}(z^2) = 0, |z| = 2$ is
(a) 4 (b) 3 (c) 2 (d) 1

2. If $\hat{a}$ and $\hat{b}$ are unit vectors such that $\hat{a}+3\hat{b}$ is perpendicular to $7\hat{a}-5\hat{b}$, then the angle between $\hat{a}$ and $\hat{b}$ is :
(a) $\frac{\pi}{3}$ (b) $\pi/6$
(c) $2\pi/3$ (d) none of these

3. In an ellipse, if the lines joining focus to the extremities of the major axis form an equilateral triangle with the minor axis, then the eccentricity of the ellipse is
(a) $\frac{\sqrt{3}}{2}$ (b) $\frac{\sqrt{3}}{4}$ (c) $\frac{1}{\sqrt{2}}$ (d) $\sqrt{\frac{2}{3}}$

4. Let A and B be any two events. Which one of the following statements is always true?
(a) $P(A'/B) = P(A/B)$ (b) $P(A'/B) = P(B'/A)$
(c) $P(A'/B) = 1-P(A/B)$ (d) $P(A'/B) = 1-P(A/B')$

5. A fair coin is tossed 100 times. The probability of getting tail an odd number of times is
(a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) 0 (d) 1

6. Let $f : R \to R$ be such that $f(1) = 3$, and $f'(1) = 6$. Then $\lim_{x\to 0}\left[\frac{f(1+x)}{f(1)}\right]^{\frac{1}{x}} =$
(a) 1 (b) $e^{\frac{1}{2}}$ (c) e^2 (d) e^3

7. If $y = \cos^{-1}(\cos x)$, then $\frac{dy}{dx}$ is
(a) 1 in the whole plane
(b) – 1 in the whole plane
(c) 1 in the 2^{nd} and 3^{rd} quadrants of the plane
(d) – 1 in the 3^{rd} and 4^{th} quadrants of the plane

8. Which one of the following is not true?
(a) $|\sin x| \leq 1$ (b) $-1 \leq \cos x \leq 1$
(c) $|\sec x| < 1$
(d) $\text{cosec } x \geq 1$ or $\text{cosec } x \leq -1$

9. If ${}^{n-1}C_r = (k^2 - 3)({}^{n}C_{r+1})$, then k belongs to
(a) $(\sqrt{3}, 2)$ (b) $(-\infty, -2)$
(c) $[-\sqrt{3}, \sqrt{3}]$ (d) $(2, \infty)$

10. If P is a point (x, y) on the line $y = -3x$ such that P and the point (3, 4) are on the opposite sides of the line $3x - 4y - 8 = 0$ then
(a) $x > \frac{8}{15}, y < -\frac{8}{5}$ (b) $x > \frac{8}{15}, y < -\frac{9}{15}$
(c) $x = \frac{8}{15}, y = -\frac{8}{5}$ (d) none of these

11. If $\tan^{-1}4x + \tan^{-1}6x = \pi/4$, then x is equal to
(a) $\frac{1}{12}$ (b) $\frac{1}{12}$ or $-\frac{1}{2}$
(c) $-\frac{1}{2}$ (d) none of these

12. The degree of the differential equation satisfying $\sqrt{1-x^2}+\sqrt{1-y^2} = a(x-y)$ is
(a) 1 (b) 3
(c) 2 (d) none of these

13. Coefficient of variations of two distributions are 55 and 65, and their standard deviations are 22 and 39 respectively. Their arithmetic means are respectively
(a) 15, 20 (b) 40, 60
(c) 30, 50 (d) none of these

14. If $f(x) = \frac{x^2-1}{x^2+1}$ for every real number x, then the minimum value of f
(a) – 1 (b) does not exist
(c) 0 (d) 1

15. Limit of $\int_0^x \left[\frac{1}{\sqrt{1+t^2}} - \frac{1}{1+t}\right] dt$ as $x \to \infty$ is
(a) $\log_2 e$ (b) $\log_e 2$
(c) $\log_2\left(\frac{1}{e}\right)$ (d) $\log_{\frac{1}{e}} 2$

16. The length L (in centimetre) of a copper rod is a linear function of its Celsius temperature C. In an experiment $L = 124.942$ when $C = 20$ and

$L = 125.134$ when $C = 110$. The expression of L in terms of C is

(a) $L = \frac{0.192}{90}(C - 20) + 124.942$

(b) $L = \frac{0.192}{90}(C - 110) + 124.942$

(c) $L = \frac{192}{90}(C - 20) + 124.942$

(d) $L = \frac{192}{90}(C - 110) + 124.942$

17. If $f(0) = 0$, $f'(0) = 3$, then $y'(0)$ will be equal to, where $y = f(f(f(f(f(x)))))$

(a) 0 (b) 3 (c) 3^4 (d) 3^5

18. The product of r consecutive integers is divisible by

(a) $r!$ (b) $(r - 1)!$

(c) $(r + 1)!$ (d) none of these

19. If a hyperbola passes through the foci of the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ and its transverse and conjugate axes coincide with the major and minor axes of the ellipse and product of their eccentricities be 1, then the equation of hyperbola is

(a) $\frac{x^2}{9} - \frac{y^2}{25} = 1$ (b) $\frac{x^2}{9} - \frac{y^2}{16} = 1$

(c) $\frac{x^2}{16} - \frac{y^2}{25} = 1$ (d) none of these

20. If z_1, z_2 and z_3 are complex numbers such that

$|z_1| = |z_2| = |z_3| = \left|\frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3}\right| = 1$,

then $|z_1 + z_2 + z_3|$ equals :

(a) 3 (b) 1

(c) 2 (d) none of these

21. Let $I_n = \int_0^1 x^n \tan^{-1} x\,dx$. If $a_n I_{n+2} + b_n I_n = c_n$ for all $n \geq 1$, then

(a) $b_1, b_2, b_3, ...$ are in A.P.

(b) $b_1, b_2, b_3, ...$ are in G.P.

(c) $b_1, b_2, b_3, ...$ are in H.P.

(d) none of these

22. If $S_1 = a_2 + a_4 + a_6 + ...$ upto 100 terms and $S_2 = a_1 + a_3 + a_5 + ...$ upto 100 terms of a certain A.P., then its common difference is

(a) $S_1 - S_2$ (b) $S_2 - S_1$

(c) $\frac{S_1 - S_2}{2}$ (d) none of these

23. The image of the point (1, 2, 3) by the plane $x + y + z + 3 = 0$ is

(a) $(-5, 4, -3)$ (b) $(-5, -4, -3)$

(c) $(5, -4, 3)$ (d) $(5, 4, 3)$

24. The equation of the plane containing the line $\frac{x - x_1}{l} = \frac{y - y_1}{m} = \frac{z - z_1}{n}$ is $a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$, where

(a) $ax_1 + by_1 + cz_1 = 1$

(b) $a/l = b/m = c/n$

(c) $lx_1 + my_1 + nz_1 = 0$

(d) $al + bm + cn = 0$

25. For the binomial distribution $(p + q)^n$, whose mean is 20 and variance is 16, pair (n, p) is

(a) $\left(100, \frac{1}{5}\right)$ (b) $\left(100, \frac{2}{5}\right)$

(c) $\left(50, \frac{1}{5}\right)$ (d) $\left(50, \frac{2}{5}\right)$

26. The value of $\begin{vmatrix} \sin\alpha & \cos\alpha & \sin(\alpha+\gamma) \\ \sin\beta & \cos\beta & \sin(\beta+\gamma) \\ \sin\delta & \cos\delta & \sin(\gamma+\delta) \end{vmatrix}$ is :

(a) $\sin\alpha \sin\beta \sin\delta$ (b) $\sin\alpha \cos\beta \cos\delta$

(c) 1 (d) 0

27 The value of $\left[(0.16)^{\log_{2.5}\left(\frac{1}{3}+\frac{1}{3^2}+\frac{1}{3^3}+.......\infty\right)}\right]^{1/2}$ is

(a) 1 (b) -1

(c) 0 (d) none of these

28. A unit vector $\vec{a}$ makes angles $\pi/4$ with $\hat{i}$, $\pi/3$ with $\hat{j}$ and an acute angle θ with $\hat{k}$, then θ and $\vec{a}$ are

(a) $\frac{\pi}{3}, \frac{\sqrt{2}\,\hat{i} + \hat{j} + \hat{k}}{2}$ (b) $\frac{\pi}{3}, \frac{\sqrt{2}\,\hat{i} - \hat{j} + \hat{k}}{2}$

(c) $\frac{\pi}{3}, \frac{\sqrt{2}\,\hat{i} + \hat{j} - \hat{k}}{2}$ (d) $\frac{\pi}{3}, \frac{\hat{i} + \hat{j} + \hat{k}}{2}$

29. Let $P = (-1, 0)$, $O = (0, 0)$ and $Q = (3, 3\sqrt{3})$ be three points. Then, the equation of the bisector of $\angle POQ$ is:

(a) $y = \sqrt{3}\,x$ (b) $\sqrt{3}\,y = x$

(c) $y = -\sqrt{3}\,x$ (d) $\sqrt{3}\,y = -x$

30. The locus of the point of intersection of the tangents at the extremeties of a chord of the circle $x^2 + y^2 = a^2$ which touches the circle $x^2 + y^2 - 2ax = 0$ passes through the point

(a) $(a/2, 0)$ (b) $(0, a/2)$

(c) $(a, 0)$ (d) $(0, 0)$

31. A variable line through the point $\left(\frac{1}{5}, \frac{1}{5}\right)$ cuts the coordinate axes in the points A and B. If the point P divides AB internally in the ratio 3 : 1, then the locus of P is
(a) $3y + x = 20xy$ (b) $y + 3x = 20xy$
(c) $x + y = 20xy$ (d) $3x + 3y = 20xy$

32. If the 4th term in the expansion of $\left(ax + \frac{1}{x}\right)^n$ is $\frac{5}{2}$, for all $x \in R$, then the values of a and n are respectively :
(a) $\frac{1}{2}$, 6 (b) $\frac{1}{2}$, 5 (c) 2, 6 (d) 2, 5

33. If $ax^2 + bx + c = 0$ and $2x^2 + 3x + 4 = 0$ have a common root where $a, b, c, \in N$ (set of natural numbers), the least value of $a + b + c$ is
(a) 13 (b) 11 (c) 7 (d) 9

34. The ratio in which the line segment joining the points (4, 8, 10) and (6, 10, –8) is divided by xy-plane is
(a) 5 : 4 externally (b) 5 : 4 internally
(c) 3 : 2 externally (d) none of these

35. The least value of the function $f(x) = ax + b/x$, $a > 0$, $b > 0$, $x > 0$ is
(a) $\sqrt{ab}$ (b) $2\sqrt{\frac{a}{b}}$ (c) $2\sqrt{\frac{b}{a}}$ (d) $2\sqrt{ab}$

36. Let R and C denote the set of real numbers and complex numbers respectively. The function $f : C \to R$ defined by $f(z) = |z|$ is
(a) one to one (b) onto
(c) bijective
(d) neither one to one nor onto

37. Let m be a positive integer and $0 \le r \le m$.
The value of $\sum_{r=0}^{m} \begin{vmatrix} 2r-1 & {}^mC_r & 1 \\ m^2-1 & 2^m & m+1 \\ \sin^2 m & \cos^2 m & \tan^2 m \end{vmatrix}$ will be
(a) 2^m (b) $m + 1$ (c) $m^2 - 1$ (d) 0

38. Let $f = \{(1, 1), (2, 4), (0, -2), (-1, -5)\}$ be a linear function from Z into Z. Then $f(x) =$
(a) $3x - 2$ (b) $6x - 8$
(c) $5x - 2$ (d) $7x + 2$

39. Let $z = 2x + 3y$, subject to constraints $y \le x - 1$
$y \ge x + 1$, $x \ge 0$, $y \ge 0$,
then the maximum of z is
(a) 2 (b) 3
(c) does not exist (d) 10

40. If $f'(x) = g(x)$ and $g'(x) = -f(x)$ for all x and $f(2) = 4 = f'(2)$, then $f^2(4) + g^2(4)$ is
(a) 8 (b) 16 (c) 32 (d) 64

41. If $I_1 = \int_0^{\pi/2} f(\sin 2x)\sin x \, dx$ and $I_2 = \int_0^{\pi/4} f(\cos 2x)\cos x \, dx$ then $I_1/I_2 =$
(a) 1 (b) $\sqrt{2}$ (c) $\frac{1}{\sqrt{2}}$ (d) 2

42. If $2^x + 2^y = 2^{x+y}$, then the value of $\frac{dy}{dx}$ at $x = y = 1$ is
(a) 0 (b) –1 (c) 1 (d) 2

43. For a natural number n, which one is the correct statement?
(a) $1^3 + 2^3 + ... + n^3 = (1 + 2 + ... + n)^2$
(b) $1^3 + 2^3 + ... + n^3 > (1 + 2 + ... + n)^2$
(c) $1^3 + 2^3 + ... + n^3 < (1 + 2 + ... + n)^2$
(d) $1^3 + 2^3 + ... + n^3 \ne (1 + 2 + ... + n)^2$

44. The angle between the tangents drawn from the point (1, 4) to the parabola $y^2 = 4x$, is :
(a) $\pi/2$ (b) $\pi/6$ (c) $\pi/4$ (d) $\pi/3$

45. If $f(x) = 3e^{x^2}$, then $f'(x) - 2x f(x) + \frac{1}{3} f(0) - f'(0)$ is equal to
(a) 0 (b) 1
(c) $\frac{7}{3}e^{x^2}$ (d) none of these

46. The area of the region bounded by the line $y = 3x + 2$, the x-axis and the ordinates $x = -1$ and $x = 1$ is
(a) $\frac{13}{3}$ (b) $\frac{13}{4}$ (c) $\frac{13}{5}$ (d) $\frac{13}{6}$

47. If $a \sin^{-1}x - b\cos^{-1}x = c$, then $a \sin^{-1}x + b \cos^{-1}x$ is equal to

(a) $\dfrac{\pi ab+c(a-b)}{a+b}$ (b) 0

(c) $\dfrac{\pi ab-c(a-b)}{a+b}$ (d) $\dfrac{\pi}{2}$

48. If the system of equations

$ax + ay - z = 0,$

$bx - y + bz = 0,$

$-x + cy + cz = 0$

has a non-trivial solution, then the value of

$\dfrac{1}{1+a}+\dfrac{1}{1+b}+\dfrac{1}{1+c}$ is

(a) 0 (b) 1 (c) 2 (d) 3

49. If ${}^{12}P_r = {}^{11}P_6 + 6\cdot{}^{11}P_5$, then $r =$

(a) 7 (b) 5 (c) 6 (d) 4

50. If $A=\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}$, then A^n is

(a) A (b) $\begin{bmatrix}3^{n-1}&3^{n-1}&3^{n-1}\\3^{n-1}&3^{n-1}&3^{n-1}\\3^{n-1}&3^{n-1}&3^{n-1}\end{bmatrix}$

(c) $\begin{bmatrix}3^{n}&3^{n}&3^{n}\\3^{n}&3^{n}&3^{n}\\3^{n}&3^{n}&3^{n}\end{bmatrix}$ (d) none of these

SOLUTIONS

1. (a) : Let $z = x + iy$

Then $z^2 = x^2 - y^2 + 2ixy$

Now $\text{Re}(z^2) = 0 \Rightarrow x^2 - y^2 = 0$

$\Rightarrow x^2 = y^2 \Rightarrow y = \pm x$

Clearly lines $y = \pm x$ intersects circle $|z| = 2$ at four points. Hence, the given system has four solutions.

2. (a) : $\hat{a}+3\hat{b}$ and $7\hat{a}-5\hat{b}$ are perpendicular to each other.

$\therefore\ (\hat{a}+3\hat{b})\cdot(7\hat{a}-5\hat{b})=0$

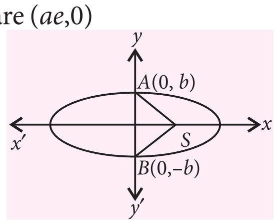
$\Rightarrow 7(\hat{a}\cdot\hat{a})-5(\hat{a}\cdot\hat{b})+21(\hat{b}\cdot\hat{a})-15(\hat{b}\cdot\hat{b})=0$

$\because\ \hat{a}\cdot\hat{b}=\hat{b}\cdot\hat{a}$ and $\hat{a}\cdot\hat{a}=\hat{b}\cdot\hat{b}=1$

$\therefore\ 16\hat{a}\cdot\hat{b}=8 \Rightarrow \hat{a}\cdot\hat{b}=\dfrac{1}{2}$

$\Rightarrow |\hat{a}|\cdot|\hat{b}|\cos\theta=\dfrac{1}{2} \Rightarrow \cos\theta=\dfrac{1}{2} \Rightarrow \theta=\dfrac{\pi}{3}$

3. (a) : In the given figure, S is focus whose coordinates are $(ae,0)$



$\because$ ΔABS is an equilateral

$\therefore$ Area of $\Delta ABS=\dfrac{1}{2}\times AB\times OS=\dfrac{\sqrt{3}}{4}(\text{Side})^2$

$\Rightarrow \dfrac{1}{2}\times 2b\times ae=\dfrac{\sqrt{3}}{4}(2b)^2 \Rightarrow ae=\sqrt{3}b$ ....(i)

Also $b^2 = a^2(1 - e^2)$

$\Rightarrow \left(\dfrac{ae}{\sqrt{3}}\right)^2=a^2(1-e^2)\Rightarrow e^2=3-3e^2 \Rightarrow e=\dfrac{\sqrt{3}}{2}$

4. (c): $P(A'/B)=\dfrac{P(A'\cap B)}{P(B)}=\dfrac{P(B)-P(A\cap B)}{P(B)}$

$=1-\dfrac{P(A\cap B)}{P(B)}=1-P(A/B).$

5. (a)

6. (c): Let $y=\left[\dfrac{f(1+x)}{f(1)}\right]^{1/x}$

$\Rightarrow \ln y=\dfrac{1}{x}\ln\left[\dfrac{f(1+x)}{f(1)}\right]=\dfrac{\ln f(1+x)-\ln 3}{x}$

$\Rightarrow \lim\limits_{x\to 0}\ln y=\lim\limits_{x\to 0}\dfrac{\ln f(1+x)-\ln 3}{x}$

$=\lim\limits_{x\to 0}\dfrac{f'(1+x)}{f(1+x)}=\dfrac{f'(1)}{f(1)}=\dfrac{6}{3}=2 \Rightarrow y=e^2$

7. (d) : $y=\cos^{-1}(\cos x)=\begin{cases}x\ ; & 0\le x\le\pi\\ 2\pi-x; & \pi\le x\le 2\pi\end{cases}$

$\therefore\ \dfrac{dy}{dx}=1$ in Ist and IInd quadrants *i.e.* $(0, \pi)$ and -1 in 3rd and 4th quadrant *i.e.* $(\pi, 2\pi)$

8. (c): The range of $\sec x$ is $(-\infty, -1] \cup [1, \infty)$

$\therefore\ |\sec x| \ge 1$

9. (a) : From given condition,

$\dfrac{(n-1)!}{r!(n-1-r)!}=(k^2-3)\cdot\dfrac{n!}{(r+1)!(n-r-1)!}$

$\Rightarrow (r + 1) = (k^2 - 3)\cdot n \Rightarrow k^2=3+\dfrac{r+1}{n}$

Now, from given equation,

$r \le n - 1$. [$\because\ {}^{n-1}C_r$ is defined for $r \le n - 1$ or ${}^nC_{r+1}$ is defined for $r + 1 \le n$]

$\Rightarrow r+1\le n \Rightarrow \dfrac{r+1}{n}\le 1$

Also, in ${}^{n-1}C_r$ and ${}^nC_{r+1}$, we have $r \ge 0$.

So, $k^2=3+\dfrac{r+1}{n} \Rightarrow 3 < k^2 \le 4 \Rightarrow k\in(\sqrt{3},2]$

10. (a) : Point (3, 4) lies on one side of $3x - 4y - 8 = 0$

$\Rightarrow 3 \times 3 - 4 \times 4 - 8 = 9 - 16 - 8 < 0$

$\therefore$ $P(x, y)$ lies on the other side of $3x - 4y - 8 = 0$ such that $3x - 4y - 8 > 0 \Rightarrow 3x - 4y - 8 > 0$

also $y = -3x \Rightarrow 3x - 4(-3x) - 8 > 0$

$\Rightarrow \quad 3x + 12x - 8 > 0 \Rightarrow 15x > 8 \Rightarrow x > \frac{8}{15}$

Also $y = -3x \Rightarrow x = -\frac{y}{3}$

$\Rightarrow 3\times\left(-\frac{y}{3}\right) - 4y - 8 > 0 \Rightarrow 5y + 8 < 0$

$\Rightarrow 5y < -8 \Rightarrow y < \frac{-8}{5} \quad \therefore \ x > \frac{8}{15}, y < -\frac{8}{5}$

11. (a) : $\tan^{-1} 6x + \tan^{-1} 4x = \frac{\pi}{4}$

$\tan^{-1}\left(\frac{6x+4x}{1-24x^2}\right) = \tan^{-1} 1$

$\Rightarrow \quad 10x = 1 - 24x^2 \Rightarrow 24x^2 + 10x - 1 = 0$

$\Rightarrow \quad (2x + 1)(12x - 1) = 0 \quad \Rightarrow \quad x = -\frac{1}{2}, \frac{1}{12}$

But $x = -\frac{1}{2}$ does not satisfy equation, so $x = \frac{1}{12}$

12. (a) : Put $x = \sin\alpha$, $y = \sin\beta$

$\therefore \quad \sqrt{1-x^2} + \sqrt{1-y^2} = a(x - y)$

becomes $\cos\alpha + \cos\beta = a(\sin\alpha - \sin\beta)$

$2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right) = a\left(2\cos\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)\right)$

$\Rightarrow \quad \cot\left(\frac{\alpha-\beta}{2}\right) = a \Rightarrow \frac{\alpha-\beta}{2} = \cot^{-1} a$

$\Rightarrow \alpha - \beta = 2\cot^{-1} a \Rightarrow \sin^{-1} x - \sin^{-1} y = 2\cot^{-1} a$

$\Rightarrow \frac{1}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-y^2}}\frac{dy}{dx} = 0$

$\therefore$ Degree of the differential equation is 1.

13. (b) : Coefficient of variation $= \frac{\sigma}{\bar{X}} \times 100$

$\therefore \quad \bar{X}_1 = \frac{22}{55}\times 100 = 40, \ \bar{X}_2 = \frac{39}{65}\times 100 = 60$

$\therefore$ Means are 40, 60

14. (a) : $f(x) = \frac{x^2-1}{x^2+1}$

$\Rightarrow \quad x^2 f(x) + f(x) = x^2 - 1 \Rightarrow x^2(f(x) - 1) = -1 - f(x)$

$\Rightarrow \quad x^2 = \frac{1+f(x)}{1-f(x)}$

Now, $x^2 \geq 0 \quad \Rightarrow \quad \frac{1+f(x)}{1-f(x)} \geq 0$

$\Rightarrow \quad (f(x) - 1)(f(x) + 1) \leq 0 \Rightarrow f(x) \in [-1, 1]$

Thus minimum value of $f(x)$ is -1.

15. (d)

16. (a) : Points (124.942, 20) ; (125.134, 110)

$\therefore$ Required equation is

$(C - 20) = \left(\frac{110-20}{125.134-124.942}\right)(L - 124.942)$

$\Rightarrow \quad (C - 20) = \left(\frac{90}{0.192}\right)(L - 124.942)$

$\Rightarrow \quad \frac{0.192}{90}(C - 20) + 124.942 = L$

(Using two point equation of a line)

17. (d) : $y = f(f(f(f(f(x)))))$

$y(x) = f(f(f(f \text{ of}(x))))$

Since $f'(0) = 3 \Rightarrow y'(0) = 3^5$

18. (a) : Let the consecutive integers be

$n(n + 1)(n + 2) \ldots (n + (r - 1))$

Product $= n(n + 1)(n + 2) \ldots (n + (r - 1))$

$= \frac{(n-1)!\,n(n+1)(n+2)\ldots(n+(r-1))\,r!}{(n-1)!\,r!}$

$= {}^{(n+r-1)}C_r \, r!$

$\therefore \quad$ Product is divisible by $r!$

19. (b) : Given ellipse is $\frac{x^2}{25} + \frac{y^2}{16} = 1$

$\therefore \quad b^2 = a^2(1-e^2) \Rightarrow \frac{16}{25} = 1 - e^2 \Rightarrow e = \frac{3}{5}$

Let equation of hyperbola be

$\frac{x^2}{a'^2} - \frac{y^2}{b'^2} = 1 \qquad \ldots\text{(i)}$

$\therefore \quad a'^2 + b'^2 = a'^2 e'^2$

Since $e \times e' = 1 \Rightarrow e' = \frac{5}{3}$

$\Rightarrow \quad a'^2 + b'^2 = \frac{25}{9}a'^2 \Rightarrow 9b'^2 = 16a'^2$

Also coordinates of focus of ellipse are

$(\pm ae, 0) = (\pm 3, 0)$

$\because$ Hyperbola passes through $(\pm 3, 0)$

$\therefore \quad \frac{9}{a'^2} = 1 \Rightarrow a'^2 = 9 \Rightarrow a' = 3 \quad$ (from (i))

$\therefore \quad$ From (ii) $9b'^2 = 16a'^2 \Rightarrow b'^2 = 16 \Rightarrow b' = 4$

$\therefore$ Equation of hyperbola is $\frac{x^2}{9} - \frac{y^2}{16} = 1$

20. (b) : $|z_1 + z_2 + z_3| = |\overline{z_1 + z_2 + z_3}| \qquad [\because |z| = |\bar{z}|]$

$= |\bar{z}_1 + \bar{z}_2 + \bar{z}_3| = \left|\frac{\bar{z}_1 z_1}{z_1} + \frac{\bar{z}_2 z_2}{z_2} + \frac{\bar{z}_3 z_3}{z_3}\right|$

$= \left|\frac{|z_1|^2}{z_1} + \frac{|z_2|^2}{z_2} + \frac{|z_3|^2}{z_3}\right| \qquad [\because z\bar{z} = |z|^2]$

$= \left|\frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3}\right| = 1$

21. (a)

22. (d) : $S_1 = a_2 + a_4 + a_6 + a_8 + \ldots + a_{100}$

$S_2 = a_1 + a_3 + a_5 + a_7 + \ldots + a_{99}$

$$S_1 - S_2 = (a_2 - a_1) + (a_4 - a_3) + \ldots + (a_{100} - a_{99})$$
$$= d + d + \ldots + d = 50d$$
$$\Rightarrow \ d = \frac{S_1 - S_2}{50}$$

23. (b) : Let image of the point P (1, 2, 3) in the given plane be Q. Equation of line through (1, 2, 3) and normal to given plane is $\frac{x-1}{1} = \frac{y-2}{1} = \frac{z-3}{1} = r$

Since this line passes through Q, so let the coordinates of Q be $(r + 1, r + 2, r + 3)$

The coordinates of mid point of PQ are

$\left(\frac{r+2}{2}, \frac{r+4}{2}, \frac{r+6}{2}\right)$ lies on plane $x + y + z + 3 = 0$

$\Rightarrow \quad r + 2 + r + 4 + r + 6 + 6 = 0 \Rightarrow r = -6$

$\therefore \quad Q$ is $(-5, -4, -3)$

24. (d)

25. (a): For binomial distribution mean $= np = 20$ ...(i)

variance $= npq = 16$ ...(ii)

$20q = 16$ (from (i) and (ii))

$$\Rightarrow \quad q = \frac{16}{20} = \frac{4}{5} \ \Rightarrow 1 - p = \frac{4}{5} \Rightarrow p = \frac{1}{5}$$

Now, $n \times \frac{1}{5} = 20 \ \Rightarrow \ n = 100$

$$\therefore \quad (n, p) = \left(100, \frac{1}{5}\right)$$

26. (d) : Let $\Delta = \begin{vmatrix} \sin\alpha & \cos\alpha & \sin(\alpha+\gamma) \\ \sin\beta & \cos\beta & \sin(\beta+\gamma) \\ \sin\delta & \cos\delta & \sin(\gamma+\delta) \end{vmatrix}$

Applying $C_3 \to C_3 - \cos\gamma\, C_1 - \sin\gamma\, C_2$, we get

$$\Delta = \begin{vmatrix} \sin\alpha & \cos\alpha & 0 \\ \sin\beta & \cos\beta & 0 \\ \sin\delta & \cos\delta & 0 \end{vmatrix} = 0$$

27. (d) : Let $y = \left[(0.16)^{\log_{2.5}\left(\frac{1}{2}\right)}\right]$

$$\left[\because \frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \ldots\infty = \frac{\frac{1}{3}}{1 - \frac{1}{3}} = \frac{1}{3} \times \frac{3}{2} = \frac{1}{2}\right]$$

$$\Rightarrow \quad \log y = \log_{2.5}(1/2)\log(0.16)$$

$$\Rightarrow \quad \log y = \frac{-\log 2}{\log(1/0.4)} \times 2\log 0.4$$

$$\log y = \frac{-\log 2}{-\log 0.4} \times 2\log 0.4 \Rightarrow \log y = 2\log 2$$

$$y = 2^2 = 4 \Rightarrow (y)^{\frac{1}{2}} = 2$$

28. (a) : $\vec{a} = x\hat{i} + y\hat{j} + z\hat{k}$

$\sqrt{x^2 + y^2 + z^2} = 1 \qquad (\because \vec{a}$ is a unit vector)

$$\vec{a}\cdot\hat{i} = |\vec{a}||\hat{i}|\cos\frac{\pi}{4} \ \Rightarrow \ \vec{a}\cdot\hat{i} = \frac{1}{\sqrt{2}} = x$$

$$\vec{a}\cdot\hat{j} = |\vec{a}||\hat{j}|\cos\frac{\pi}{3} \ \Rightarrow \ y = \frac{1}{2}$$

$$\vec{a}\cdot\hat{k} = z = \cos\theta$$

$$\therefore \quad \vec{a} = \frac{1}{\sqrt{2}}\hat{i} + \frac{1}{2}\hat{j} + \cos\theta\hat{k}$$

Since $x^2 + y^2 + z^2 = 1 \Rightarrow \frac{1}{2} + \frac{1}{4} + \cos^2\theta = 1$

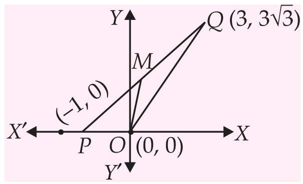
$$\cos^2\theta = 1 - \frac{3}{4} = \frac{1}{4} \Rightarrow \cos\theta = \frac{1}{2} \ \Rightarrow \theta = \frac{\pi}{3}$$

$$\therefore \quad \cos\theta = \frac{1}{2}$$

$$\vec{a} = \frac{1}{\sqrt{2}}\hat{i} + \frac{1}{2}\hat{j} + \frac{1}{2}\hat{k} = \frac{1}{2}(\sqrt{2}\,\hat{i} + \hat{j} + \hat{k})$$

$$\therefore \quad \vec{a} = \frac{1}{2}(\sqrt{2}\,\hat{i} + \hat{j} + \hat{k}) \ \text{and} \ \theta = \frac{\pi}{3}$$

29. (c): Let the bisector of $\angle POQ$ intersects the line PQ at M.



$OP = 1$,

$$OQ = \sqrt{(3\sqrt{3})^2 + 3^2} = 6$$

Now, $\frac{PM}{QM} = \frac{OP}{OQ} = \frac{1}{6}$

$\Rightarrow \quad PM : QM = 1 : 6$

Let us use section formula.

Coordinates of M are

$$\left(\frac{-6+3}{1+6}, \frac{0+3\sqrt{3}}{1+6}\right) \ i.e., \ \left(-\frac{3}{7}, \frac{3\sqrt{3}}{7}\right)$$

Equation of OM is given by

$$y - 0 = \frac{\frac{3\sqrt{3}}{7} - 0}{-\frac{3}{7} - 0}(x - 0) \Rightarrow \ y = -\sqrt{3}x$$

30. (a) : Let the point of intersection of tangents be (h, k). Then, the chord of contact is,

$xh + yk = a^2$ ... (1)

But the above chord touches the circle,

$x^2 + y^2 - 2ax = 0$

Centre is $(a, 0)$. Radius $= a$

Thus, perpendicular from the centre on line = Radius of circle.

$$\Rightarrow \quad \frac{h(a) + 0 - a^2}{\sqrt{h^2 + k^2}} = \pm a.$$

Squaring both sides, we get

$\Rightarrow \quad (h - a)^2 = (h^2 + k^2) \Rightarrow k^2 + 2ah = a^2$

So, equation of locus is, $y^2 + 2ax = a^2$

Above curve clearly passes through $(a/2, 0)$.

31. (b) : Equation to line in intercept form

$$\frac{x}{a}+\frac{y}{b}=1. \text{ It passes through } \left(\frac{1}{5},\frac{1}{5}\right)$$

$$\Rightarrow \quad a+b=5ab \qquad \text{...(i)}$$

Point $P(x,y)$ divides AB joining $A(a, 0)$ and $B(0, b)$ internally in ratio 3 : 1

$$\Rightarrow \quad 4=\frac{a}{4},\ y=\frac{3b}{4}\Rightarrow a=4x,\ b=\frac{4y}{3}$$

Keeping values of a and b in eq. (i) we get

$$4x+\frac{4y}{3}=5(4x)\left(\frac{4y}{3}\right)\Rightarrow 3x+y=20xy$$

32. (a) : $T_{r+1}={}^nC_r(ax)^{n-r}\left(\frac{1}{x}\right)^r$

For 4th term put $r = 3$

$$T_4={}^nC_3(ax)^{n-3}\left(\frac{1}{x}\right)^3 \Rightarrow \frac{5}{2}={}^nC_3\,a^{n-3}\,x^{n-6}$$

Let us put $n - 6 = 0$ to discard x.

$$\frac{5}{2}={}^6C_3\,a^3 \Rightarrow a=\frac{1}{2}$$

Thus, $a=\frac{1}{2},\ n=6$

33. (d)

34. (b) : Let xy-plane divides the join of points (4, 8, 10) and (6, 10, –8) in the ratio $\lambda : 1$

$$\frac{10-8\lambda}{\lambda+1}=0 \Rightarrow 10-8\lambda=0 \Rightarrow \lambda=\frac{5}{4}$$

35. (d) : $f(x)=ax+\frac{b}{x} \Rightarrow f'(x)=a-\frac{b}{x^2}$

For maxima or minima, $f'(x) = 0$

$$\Rightarrow \quad a=\frac{b}{x^2}\Rightarrow x=\sqrt{\frac{b}{a}}\Rightarrow f''(x)=\frac{2b}{x^3}$$

$$f''\left(\sqrt{\frac{b}{a}}\right)=\frac{2b}{\left(\frac{b}{a}\right)^{3/2}}>0$$

Hence $f(x)$ is minimum at $x=\sqrt{\frac{b}{a}}$ and minimum

value is $a\times\sqrt{\frac{b}{a}}+\frac{b}{\left(\sqrt{\frac{b}{a}}\right)}=\sqrt{ab}+\sqrt{ab}=2\sqrt{ab}$

36. (d) : $f : C \to R,\ f(z) = |z|$

Let $z_1 = 3 + 4i,\ z_2 = 4 + 3i$

$$f(z_1)=|3+4i|=\sqrt{9+16}=5$$
$$f(z_2)=|4+3i|=\sqrt{16+9}=5$$

$\Rightarrow \quad f(z_1)=f(z_2)$ but $z_1\neq z_2$

as $Re(z_1)\neq Re(z_2)$ and $Im(z_1)\neq Im(z_2)$

$\therefore f(z)$ is not one-one

$\because \quad f(z)=|z|=\sqrt{x^2+y^2}\geq 0$

$\therefore$ Range of $f(z)$ is $[0,\infty)$, while codomain is set of real numbers R *i.e.* $(-\infty, \infty)$, so range $\neq$ codomain

$\therefore \ f(z)$ is not onto.

37. (d) : Now, $\displaystyle\sum_{r=0}^{m}\begin{vmatrix} 2r-1 & {}^mC_r & 1 \\ m^2-1 & 2^m & m+1 \\ \sin^2 m & \cos^2 m & \tan^2 m \end{vmatrix}$

$$=\begin{vmatrix} \sum_{r=0}^{m}(2r-1) & \sum_{r=0}^{m}{}^mC_r & \sum_{r=0}^{m}1 \\ m^2-1 & 2^m & m+1 \\ \sin^2 m & \cos^2 m & \tan^2 m \end{vmatrix}$$

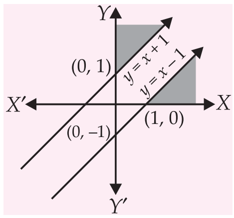
$$=\begin{vmatrix} m(m+1)-m-1 & 2^m & m+1 \\ m^2-1 & 2^m & m+1 \\ \sin^2 m & \cos^2 m & \tan^2 m \end{vmatrix}$$

$$=\begin{vmatrix} m^2-1 & 2^m & m+1 \\ m^2-1 & 2^m & m+1 \\ \sin^2 m & \cos^2 m & \tan^2 m \end{vmatrix}=0 \quad [\because R_1 \text{ and } R_2 \text{ are identical}]$$

38. (a) : $f = \{(1, 1), (2, 4), (0, –2), (–1, –5)\}$ be a linear function from Z to Z only function satisfying the above points if $f(x) = 3x – 2$

39. (c): There is no common region to fulfil the given conditions.

$\therefore$ No maximum value of z exists.



40. (c): Let $h(x) = f^2(x) + g^2(x)$

$h'(x) = 2f(x)f'(x) + 2g(x)g'(x)$

$= 2f(x)f'(x) – 2f(x)f'(x) = 0$

$\Rightarrow \quad h(x)$ = constant .... (i)

$h(2) = (f(2))^2 + (g(2))^2 = (f(2))^2 + (f'(2))^2$

$= 4^2 + 4^2 = 32\ (f(2) = f'(2) = 4)$

From (i) $h(2) = 32$ = constant

$\therefore \quad h(x) = 32 \quad \therefore \quad f^2(x) + g^2(x) = 32$

Then $f^2(4) + g^2(4) = 32$

41. (b): $I_1=\displaystyle\int_0^{\pi/2} f(\sin 2x)\sin x\,dx$ ... (i)

$$= \int_0^{\pi/2} f\left(\sin 2\left(\frac{\pi}{2}-x\right)\right)\cdot \sin\left(\frac{\pi}{2}-x\right)dx$$

$$= \int_0^{\pi/2} f(\sin 2x)\cos x\,dx \quad \text{... (ii)}$$

Adding (i) and (ii), we get

$$2I_1 = \int_0^{\pi/2} f(\sin 2x)(\sin x + \cos x)\,dx$$

$$= \sqrt{2}\int_0^{\pi/2} f(\sin 2x)\sin\left(x+\frac{\pi}{4}\right)dx$$

put $x+\frac{\pi}{4} = \left(\frac{\pi}{2}-\theta\right)$ $\quad$ (*i.e.*, $x = \frac{\pi}{4}-\theta$)

$$\Rightarrow\ 2I_1 = -\sqrt{2}\int_{\pi/4}^{-\pi/4} f(\cos 2\theta)\cos\theta\,d\theta$$

$$= \sqrt{2}\int_{-\pi/4}^{\pi/4} f(\cos 2\theta)\cos\theta\,d\theta = 2\sqrt{2}\int_0^{\pi/4} f(\cos 2\theta)\cos\theta\,d\theta$$

$$\Rightarrow\ I_1 = \sqrt{2}\int_0^{\pi/4} f(\cos 2\theta)\cos\theta\,d\theta$$

$$I_2 = \int_0^{\pi/4} f(\cos 2\theta)\cos\theta\,d\theta,\ \Rightarrow \frac{I_1}{I_2} = \frac{\sqrt{2}}{1}.$$

42. (b) : $2^x + 2^y = 2^{x+y}$ ... (1)

Differentiating with respect to x, we get

$$2^x\log 2 + 2^y\log 2\cdot\frac{dy}{dx} = 2^{x+y}\log 2\left(1+\frac{dy}{dx}\right) \text{... (2)}$$

At $x = y = 1$, (2) becomes

$$2^1\log 2 + 2^1\log 2\frac{dy}{dx} = 2^2\log 2\left(1+\frac{dy}{dx}\right)$$

$$\Rightarrow\ 2\log 2 - 2^2\log 2 = 4\log 2\frac{dy}{dx} - 2\log 2\frac{dy}{dx}$$

$$\Rightarrow\ 2\log 2 - 4\log 2 = \frac{dy}{dx}(4\log 2 - 2\log 2)$$

$$\Rightarrow\ \frac{dy}{dx} = -1$$

43. (a) : $1^3 + 2^3 + ... + n^3$

$$= \left(\frac{n(n+1)}{2}\right)^2 = (1+2+......+n)^2$$

44. (d)

45. (b) : $f(x) = 3e^{x2}$

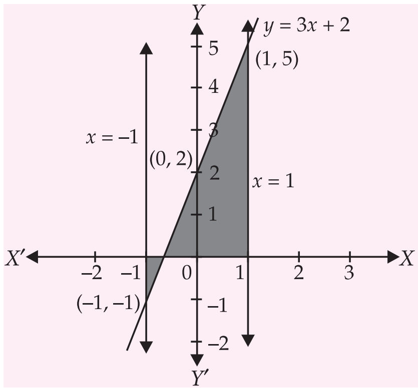
$f'(x) = 3e^{x^2}\times 2x = 6xe^{x^2}$

$f(0) = 3, f'(0) = 0$

$$f'(x) - 2xf(x) + \frac{1}{3}f(0) - f'(0)$$

$$= 6xe^{x^2} - 2x\times 3e^{x^2} + \frac{1}{3}\times 3 - 0 = 6xe^{x^2} - 6xe^{x^2} + 1 = 1$$

46. (a) :



$$-\int_{-1}^{-2/3}(3x+2)\,dx + \int_{-2/3}^{1}(3x+2)\,dx$$

$$= -\left[\frac{3x^2}{2}+2x\right]_{-1}^{-2/3} + \left[\frac{3x^2}{2}+2x\right]_{-2/3}^{1}$$

$$= \left(\frac{-3}{2}\left[\frac{4}{9}-1\right]+2\left[\frac{-2}{3}+1\right]\right) + \frac{3}{2}\left[1-\frac{4}{9}\right]+2\left[1+\frac{2}{3}\right]$$

$$= \left(\frac{-3}{2}\left[\frac{-5}{9}\right]+2\left[\frac{1}{3}\right]\right) + \frac{3}{2}\left[\frac{5}{9}\right]+2\left[\frac{5}{3}\right]$$

$$= -\left(-\frac{15}{18}+\frac{2}{3}\right)+\frac{15}{18}+\frac{10}{3} = -\left(-\frac{5}{6}+\frac{2}{3}\right)+\frac{5}{6}+\frac{10}{3}$$

$$= \frac{1}{6}+\frac{25}{6} = \frac{26}{6} = \frac{13}{3}$$

47. (a) : $a\sin^{-1}x - b\cos^{-1}x = c$ [Given] ... (1)

Also, $\sin^{-1}x + \cos^{-1}x = \pi/2$ ... (2)

Using (1) and (2), we get, $(-b-a)\cos^{-1}x = c - a\pi/2$

$$\Rightarrow\ \cos^{-1}x = \frac{(a\cdot\pi)/2 - c}{a+b}$$

Also, $\sin^{-1}x = \frac{\pi}{2} - \frac{\frac{a\pi}{2}-c}{a+b}$ [From (2)]

$= a\sin^{-1}x + b\cos^{-1}x$

$$= \frac{a\pi}{2} - \frac{(a^2\pi)/2 - ca}{a+b} + \frac{ab.\pi/2 - bc}{a+b}$$

$$= \frac{a(a+b)\pi - (a^2\pi - 2ca) + ab\cdot\pi - 2bc}{2(a+b)}$$

$$= \frac{ab\pi + 2ca + ab\pi - 2bc}{2(a+b)} = \frac{\{2\pi ab + 2c(a-b)\}}{2(a+b)}$$

$$= \frac{\pi ab + c(a-b)}{a+b}$$

48. (c)

49. (c): ${}^{12}P_r = {}^{11}P_6 + 6\cdot{}^{11}P_5 \Rightarrow r = 6$

$$\left(\because\ {}^nP_r = {}^{(n-1)}P_r + r\cdot{}^{(n-1)}P_{(r-1)}\right).$$

50. (b)

FULL LENGTH PRACTICE PAPER

BITSAT

Exam date : 14th to 28th May 2016

PHYSICS

1. Let the resultant of two vectors $\vec{A}$ and $\vec{B}$ be $\vec{R}$. Let the angle between $\vec{A}$ and $\vec{R}$ be α and the angle between $\vec{B}$ and $\vec{R}$ be β. Let the magnitudes of $\vec{A}$, $\vec{B}$ and $\vec{R}$ be represented by A, B and R respectively. Which of the following statements is not correct?

(a) $\dfrac{R}{\sin(\alpha+\beta)} = \dfrac{A}{\sin\alpha} = \dfrac{B}{\sin\beta}$

(b) $R\sin\alpha = B\sin(\alpha + \beta)$

(c) $A\sin\alpha = B\sin\beta$

(d) $R\sin\beta = A\sin(\alpha + \beta)$

2. A block of mass $m = 1$ kg moving on a horizontal surface with speed $v_i = 2$ m s^{-1} enters a rough patch ranging from $x = 0.10$ m to $x = 2.01$ m. The retarding force F_r on the block in this range is inversely proportional to x over this range,

$$F_r = -\frac{k}{x} \text{ for } 0.1 < x < 2.01 \text{ m}$$

$$= 0 \text{ for } x < 0.1 \text{ m and } x > 2.01 \text{ m}$$

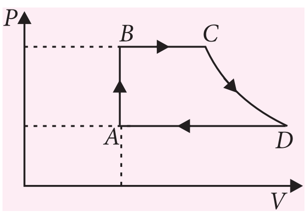
where $k = 0.5$ J. What is the final kinetic energy of the block as it crosses this patch?

(a) 0.5 J (b) 1.5 J
(c) 2.0 J (d) 2.5 J

3. A satellite is in an elliptic orbit around the earth with aphelion of $6R_E$ and perihelion of $2R_E$, where R_E is the radius of the earth.
The eccentricity of the orbit is

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) $\frac{1}{6}$

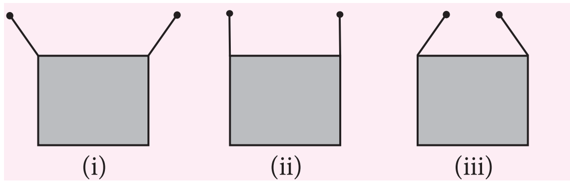
4. A cycle followed by an engine made of one mole of an ideal gas in a cylinder with a piston is shown in figure. The heat exchanged by the engine with the surroundings at constant volume is $(C_V = \frac{3}{2}R)$



(a) $(P_B - P_A)V_A$ (b) $\frac{1}{2}(P_B - P_A)V_A$

(c) $\frac{3}{2}(P_B - P_A)V_A$ (d) $\frac{5}{2}(P_B - P_A)V_A$

5. A rectangular frame is to be suspended symmetrically by two strings of equal length on two supports as shown in the figure. It can be done in one of the following three ways



The tension in the strings will be

(a) the same in all cases
(b) least in (i)
(c) least in (ii)
(d) least in (iii)

6. The radii of the two columns in U tube are r_1 and r_2. When a liquid of density ρ (angle of contact is 0°) is filled in it, the level difference of liquid in two arms is h. The surface tension of liquid is (g = acceleration due to gravity)

(a) $\dfrac{\rho g h r_1 r_2}{2(r_2 - r_1)}$ (b) $\dfrac{\rho g h(r_2 - r_1)}{2 r_1 r_2}$

(c) $\dfrac{2(r_2 - r_1)}{\rho g h r_1 r_2}$ (d) $\dfrac{\rho g h}{2(r_2 - r_1)}$

7. Two metal spheres of radii 0.01 m and 0.02 m are given a charge of 15 mC and 45 mC respectively. They are then connected by a wire. The final charge on the first sphere is ________ $\times 10^{-3}$ C.

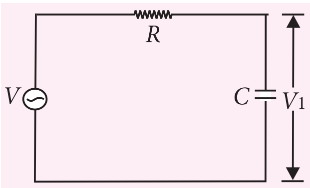
(a) 40 (b) 30 (c) 20 (d) 10

8. Magnetic field at the centre of a circular loop of area A is B. The magnetic moment of the loop will be

(a) $\frac{BA^2}{\mu_0\pi}$ (b) $\frac{BA^{3/2}}{\mu_0\,\pi}$

(c) $\frac{BA^{3/2}}{\mu_0\pi^{1/2}}$ (d) $\frac{2\,BA^{3/2}}{\mu_0\pi^{1/2}}$

9. In the circuit shown, the voltage V_1, across capacitor C



(a) is in phase with the source voltage V
(b) leads the source voltage V by 90°
(c) leads the source voltage V by an angle between 0° and 90°
(d) lags behind the source voltage V by an angle between 0° and 90°

10. The electric field (in N C^{-1}) in an electromagnetic wave is given by $E = 50\sin\omega\left(t-\frac{x}{c}\right)$. The energy stored in a cylinder of cross-section 10 cm^2 and length 100 cm along the x-axis will be

(a) 5.5×60^{-12} J (b) 1.1×10^{-11} J
(c) 2.2×10^{-11} J (d) 1.65×10^{-11} J

11. A ray incident at a point at an angle of incidence of 60° enters a glass sphere of refractive index $\mu = \sqrt{3}$ and is reflected and refracted at the further surface of the sphere. The angle between the reflected and refracted rays at this surface is

(a) 50° (b) 60° (c) 90° (d) 40°

12. Diameter of a plano-convex lens is 6 cm and thickness at the centre is 3 mm. If the speed of light in the material of the lens is 2×10^8 metre per sec, the focal length of the lens is

(a) 15 cm (b) 20 cm (c) 30 cm (d) 10 cm

13. The horizontal range of a projectile fired at an angle of 15° is 50 m. If it is fired with the same speed at an angle of 45°, its range will be

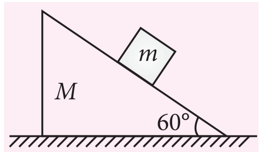
(a) 60 m (b) 71 m (c) 100 m (d) 141 m

14. A physical quantity P is related to four observables a, b, c and d as follows : $P = \frac{a^3b^2}{\sqrt{cd}}$

The percentage errors of measurement in a, b, c and d are 1%, 3%, 4% and 2%, respectively. What is the percentage error in the quantity P?

(a) 12% (b) 13% (c) 15% (d) 16%

15. In the arrangement shown in figure wedge of a mass M moves towards left with an acceleration a. All surfaces are smooth. The acceleration of mass m relative to wedge is

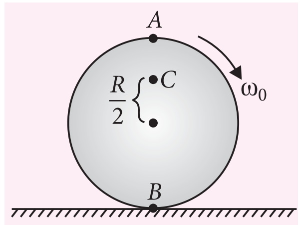


(a) $\frac{a}{2}$

(b) $\frac{2Ma}{m}$

(c) $\frac{2(M+m)a}{m}$ (d) $\frac{(M+m)a}{m}$

16. A disc rotating about its axis with angular speed ω_0 is placed lightly (without any translational push) on a perfectly frictionless table. The radius of the disc is R.



Let v_A, v_B and v_C be the magnitudes of linear velocities of the points A, B and C on the disc as shown. Then

(a) $v_A > v_B > v_C$ (b) $v_A < v_B < v_C$
(c) $v_A = v_B < v_C$ (d) $v_A = v_B > v_C$

17. The pressure and density of a diatomic gas $\left(\gamma = \frac{7}{5}\right)$ changes adiabatically from (P, d) to (P', d'). If $\frac{d'}{d} = 32$ then $\frac{P'}{P}$ is

(a) $\frac{1}{128}$ (b) 32 (c) 128 (d) 256

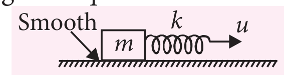
18. The period of oscillation of a mass M suspended from a spring of negligible mass is T. If along with it another mass M is also suspended, the period of oscillation will now be

(a) T (b) $\frac{T}{\sqrt{2}}$ (c) $2T$ (d) $\sqrt{2}T$

19. A copper wire 2 m long is stretched by 1 mm. If the energy stored in the stretched wire is converted to heat, find the rise in temperature of the wire. (Given $Y = 12 \times 10^{11}$ dyne cm^{-2}, density of copper = 9 g cm^{-3} and specific heat of copper = 0.1 cal g^{-1} °C^{-1})

(a) 252°C (b) (1/252)°C
(c) 1000°C (d) 2000°C

20. A block attached with a spring is kept on a smooth horizontal surface. Now the free end of the spring is pulled with a constant velocity u horizontally. Then the maximum energy stored in the spring and block system during subsequent motion is



(a) $\frac{1}{2}mu^2$

(b) mu^2

(c) $2mu^2$

(d) $4mu^2$

21. Electrons with de Broglie wavelength λ fall on the target in an X-ray tube. The cut-off wavelength of the emitted X-rays is

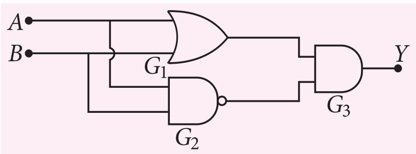
(a) $\lambda_0 = \dfrac{2mc\lambda^2}{h}$ (b) $\lambda_0 = \dfrac{2h}{mc}$

(c) $\lambda_0 = \dfrac{2m^2c^2\lambda^3}{h^2}$ (d) $\lambda_0 = \lambda$

22. Two radioactive sources A and B of half lives 1 h and 2 h respectively initially contain the same number of radioactive atoms. At the end of two hours, their rates of disintegration are in the ratio of

(a) 1 : 4 (b) 1 : 3 (c) 1 : 2 (d) 1 : 1

23. The following configuration of gates is equivalent to



(a) NAND (b) XOR

(c) OR (d) AND

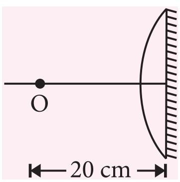
24. The period of oscillation of a freely suspended bar magnet is 4 s. If it is cut into two equal parts in length, then the time period of each part will be

(a) 4 s (b) 2 s (c) 0.5 s (d) 0.25 s

25. 100 g of an iron ball having velocity 10 m s^{-1} collides with wall at an angle 30° and rebounds with the same angle. If the period of contact between the ball and wall is 0.1 s, then the average force experienced by the wall is

(a) 10 N (b) 100 N (c) 1.0 N (d) 0.1 N

26. A point object is placed at a distance of 20 cm from a thin planoconvex lens of focal length 15 cm. The plane surface of the lens is now silvered. The image formed by the system is at



(a) 60 cm to the left of the system

(b) 60 cm to the right of the system

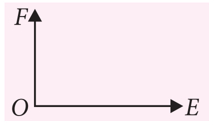
(c) 12 cm to the left of the system

(d) 12 cm to the right of the system

27. A common emitter amplifier has a voltage gain of 50, an input impedance of 100 Ω and an output impedance of 200 Ω. The power gain of the amplifier is

(a) 500 (b) 1000 (c) 1250 (d) 100

28. A body of mass 2 kg has an initial velocity of 3 m s^{-1} along OE and it is subjected to a force of 4 N in OF direction perpendicular to OE. The distance of the body from O after 4 s will be



(a) 12 m

(b) 20 m

(c) 28 m

(d) 48 m

29. A solid cylinder rolls without slipping down an inclined plane of height h. The velocity of the cylinder when it reaches the bottom is

(a) $\sqrt{\dfrac{2gh}{3}}$ (b) $\sqrt{\dfrac{4gh}{3}}$

(c) $\sqrt{\dfrac{3gh}{2}}$ (d) $\sqrt{gh}$

30. A battery of emf 8 V with internal resistance 0.5 Ω is being charged by a 120 V d.c. supply using a series resistance of 15.5 Ω. The terminal voltage of the battery is

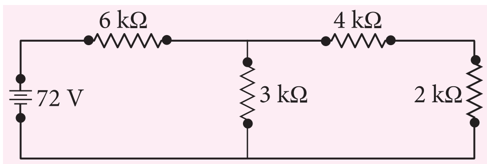
(a) 20.5 V (b) 15.5 V

(c) 11.5 V (d) 2.5 V

31. The maximum vertical distance through which a full dressed astronaut can jump on the earth is 0.5 m. Estimate the maximum vertical distance through which he can jump on the moon, which has a mean density 2/3$^{\text{rd}}$ that of earth and radius one quarter that of the earth.

(a) 1.5 m (b) 3 m (c) 6 m (d) 7.5 m

32. What current will flow through the 2 kΩ resistor in the circuit shown in the figure?



(a) 3 mA (b) 6 mA

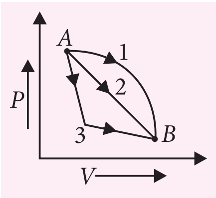
(c) 12 mA (d) 36 mA

33. In a certain place, the vertical component of earth's magnetic field is 0.5 oersted and dip angle is 60°. The earth's magnetic field at that place is

(a) 1 oersted (b) $\frac{\sqrt{3}}{2}$ oersted

(c) 2 oersted (d) $\frac{1}{\sqrt{3}}$ oersted

34. An ideal gas goes from state A to state B via three different processes as indicated in the P-V diagram.



If Q_1, Q_2, Q_3 indicate the heat absorbed by the gas along the three processes and ΔU_1, ΔU_2, ΔU_3 indicate the change in internal energy along the three processes respectively, then

(a) $Q_1 > Q_2 > Q_3$ and $\Delta U_1 = \Delta U_2 = \Delta U_3$
(b) $Q_3 > Q_2 > Q_1$ and $\Delta U_1 = \Delta U_2 = \Delta U_3$
(c) $Q_1 = Q_2 = Q_3$ and $\Delta U_1 > \Delta U_2 > \Delta U_3$
(d) $Q_3 > Q_2 > Q_1$ and $\Delta U_1 > \Delta U_2 > \Delta U_3$

35. In R-L-C series circuit, the potential differences across each element is 20 V. Now the value of the resistance alone is doubled, then potential difference across R, L and C respectively

(a) 20 V, 10 V, 10 V (b) 20 V, 20 V, 20 V
(c) 20 V, 40 V, 40 V (d) 10 V, 20 V, 20 V

36. If both the length of an antenna and the wavelength of the signal to be transmitted are doubled, the power radiated by the antenna

(a) is doubled (b) is halved
(c) is quadrupled (d) remains constant

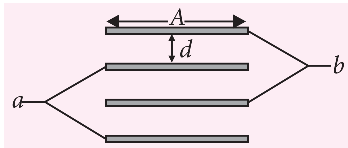
37. Two circular coils are made of two identical wires of same length and carry same current. If the number of turns of two coils are 4 and 2, then the ratio of magnetic induction at the centres will be

(a) 4 : 1 (b) 2 : 1
(c) 1 : 2 (d) 1 : 1

38. Two pendulums X and Y of time periods 4 s and 4.2 s are made to vibrate simultaneously. They are initially in phase. After how many vibrations of X, they will be in the same phase again.

(a) 30 (b) 25
(c) 21 (d) 26

39. In figure, four parallel capacitors of equal area A and spacing d are arranged, then effective capacitance between points a and b is



(a) $\frac{\varepsilon_0 A}{d}$ (b) $\frac{2\varepsilon_0 A}{d}$ (c) $\frac{3\varepsilon_0 A}{d}$ (d) $\frac{4\varepsilon_0 A}{d}$

40. An irregular closed loop carrying a current has a shape such that the entire loop cannot lie in a single plane. If this is placed in a uniform magnetic field, the force acting on the loop

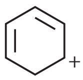
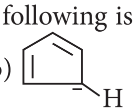
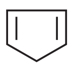
(a) must be zero (b) can never be zero
(c) may be zero
(d) will be zero only for one particular direction of the magnetic field

CHEMISTRY

41. Bond order of the species O_2, O_2^+, O_2^{2+} and O_2^{2-} increases in the order

(a) $O_2 < O_2^+ < O_2^{2+} < O_2^{2-}$
(b) $O_2^{2-} < O_2 < O_2^+ < O_2^{2+}$
(c) $O_2^+ < O_2^{2+} < O_2 < O_2^{2-}$
(d) $O_2^{2+} < O_2^+ < O_2 < O_2^{2-}$

42. Which of the following is aromatic?

(a)  (b)  (c)  (d) 

43. Which one of the following is incorrect?

(a) Boron halides are all monomeric while those of Al are dimeric.
(b) Boron halides and aluminium halides exist as monomeric halides.
(c) Boron halides and aluminium halides are Lewis acids.
(d) B_2O_3 alone is acidic while Al_2O_3 is amphoteric.

44. The wavelength of radiation emitted when an electron in a hydrogen atom makes a transition from an energy level with $n = 3$ to a level with $n = 2$ is

[Given that $E_n = \frac{-1312}{n^2}$ kJ mol^{-1}]

(a) 6.56×10^{-7} m (b) 65.6 nm
(c) 65.6×10^{-7} m (d) none of these.

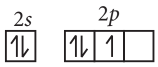
45. Which of the following is an amphoteric oxide?

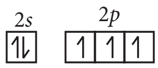
(a) CrO_3 (b) Cr_2O_3 (c) V_2O_3 (d) TiO

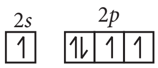
46. The correct order for the wavelength of absorption in the visible region is
(a) $[Ni(NO_2)_6]^{4-} < [Ni(NH_3)_6]^{2+} < [Ni(H_2O)_6]^{2+}$
(b) $[Ni(NO_2)_6]^{4-} < [Ni(H_2O)_6]^{2+} < [Ni(NH_3)_6]^{2+}$
(c) $[Ni(H_2O)_6]^{2+} < [Ni(NH_3)_6]^{2+} < [Ni(NO_2)_6]^{4-}$
(d) $[Ni(NH_3)_6]^{2+} < [Ni(H_2O)_6]^{2+} < [Ni(NO_2)_6]^{4-}$

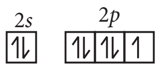
47. Consider three hypothetical ionic compounds AB, A_2B and A_2B_3 where in all the compounds, B is in –2 oxidation state and A has a variable oxidation state. What is the correct order of lattice energy of these compounds?
(a) $A_2B > AB > A_2B_3$ (b) $A_2B_3 > AB > A_2B$
(c) $AB > A_2B > A_2B_3$ (d) $A_2B_3 > A_2B > AB$

48. In which of the following compounds the carbon marked with asterisk is expected to have highest positive charge?
(a) $^*CH_3-CH_2-Cl$ (b) $^*CH_3-CH_2-Mg^+Cl^-$
(c) $^*CH_3-CH_2-Br$ (d) $^*CH_3-CH_2-CH_3$

49. The orbital diagram in which Aufbau principle is violated is
(a) 

(b) 

(c) 

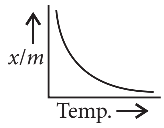
(d) 

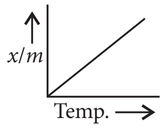
50. X and Y are two elements which form X_2Y_3 and X_3Y_4. If 0.20 mol of X_2Y_3 weighs 32.0 g and 0.4 mol of X_3Y_4 weighs 92.8 g, the atomic weights of X and Y are respectively
(a) 16.0 and 56.0 (b) 8.0 and 28.0
(c) 56.0 and 16.0 (d) 28.0 and 8.0

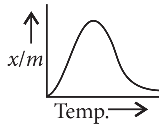
51. Bakelite is a product of the reaction between
(a) formaldehyde and NaOH
(b) aniline and urea
(c) phenol and methanal
(d) phenol and chloroform.

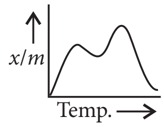
52. Which of the following is the correct order of increasing oxidizing character of oxoacids of chlorine?
(a) $HClO_3 < HClO_4 < HClO_2 < HClO$
(b) $HClO_4 < HClO_3 < HClO_2 < HClO$
(c) $HClO < HClO_4 < HClO_3 < HClO_2$
(d) $HClO < HClO_2 < HClO_3 < HClO_4$

53. Which of the following is least reactive to nitration?
(a) Benzene (b) Nitrobenzene
(c) Chlorobenzene (d) Aniline

54. Which of the following represents physical adsorption?
(a) 

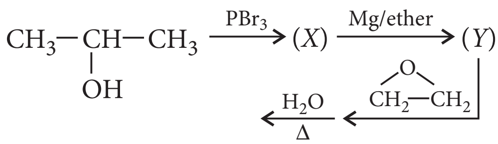
(b) 

(c) 

(d) 

55. Which of the following processes is used in the extractive metallurgy of magnesium?
(a) Fused salt electrolysis
(b) Self-reduction
(c) Aqueous solution electrolysis
(d) Thermite reduction

56. Which of the following does not represent the correct order of the property indicated?
(a) $Sc^{3+} > Cr^{3+} > Fe^{3+} > Mn^{3+}$: Ionic radii
(b) $Sc < Ti < Cr < Mn$: Density
(c) $Mn^{2+} > Ni^{2+} < Co^{2+} < Fe^{2+}$: Ionic radii
(d) $FeO < CaO > MnO > CuO$: Basic nature

57. 

The final product is
(a) $CH_3-\underset{\underset{CH_3}{|}}{CH}-CH_2CH_2OH$
(b) $CH_3-O-\underset{\underset{CH_3}{|}}{CH}-CH_2-CH_3$
(c) $CH_3-\underset{\underset{CH_3}{|}}{CH}-O-CH_2-CH_3$
(d) none of these

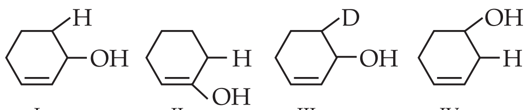
58. 100 mL of 0.1 N hypo decolourised iodine by the addition of x g of crystalline copper sulphate to excess of KI. The value of 'x' is
(Molecular wt. of $CuSO_4\cdot 5H_2O$ is 250).
(a) 5.0 g (b) 1.25 g (c) 2.5 g (d) 4 g

59. The vapour pressure of benzene at a certain temperature is 640 mm of Hg. A non-volatile and non-electrolyte solid weighing 2.175 g is added to 39.08 g of benzene. The vapour pressure of the solution is 600 mm of Hg. What is the molecular weight of solid substance?
(a) 59.5 (b) 69.5 (c) 79.6 (d) 79.9

60. Chemical A is used for water softening to remove temporary hardness. A reacts with sodium carbonate to generate caustic soda. When CO_2 is bubbled through a solution of A, it turns cloudy. What is the chemical formula of A?
(a) $CaCO_3$ (b) CaO
(c) $Ca(OH)_2$ (d) $Ca(HCO_3)_2$

61. A certain first order reaction has a rate constant of $1.0 \times 10^{-3}\ s^{-1}$ at 25°C. If the reaction rate doubles when the temperature increased to 35°C the activation energy for this reaction is
(a) 17 kJ/mol (b) 25 kJ/mol
(c) 53 kJ/mol (d) 36 kJ/mol

62. The ease of dehydration in the following compounds is



I II III IV

(a) I > III > IV > II
(b) II > I > III > IV
(c) IV > I > III > II
(d) III > I > II > IV

63. If the cell voltage is 1.23 V for the $H_2 - O_2$ fuel cell and for the half cell
$O_2 + 2H_2O + 4e^- \rightleftharpoons 4OH^-$, $E° = 0.40$ V
Then $E°$ for $H_2O + 2e^- \rightleftharpoons H_2 + 2OH^-$ will be
(a) 0.41 V (b) 0.83 V
(c) – 0.41 V (d) – 0.83 V

64. 0.001 mole of $Cr(NH_3)_5(NO_3)SO_4$ was passed through cation exchanger and the acid coming out of it required 20 mL of 0.1 M NaOH for neutralization. Hence the complex is
(a) $[Cr(NH_3)_5NO_3](SO_4)$
(b) $[Cr(NH_3)_5SO_4](NO_3)$
(c) $[Cr(NH_3)_4(NO_3)(SO_4)](NH_3)$
(d) $[Cr(NH_3)_5](NO_3)(SO_4)$

65. The principal products obtained on heating iodine with concentrated caustic soda solution are
(a) $NaOI + NaI$
(b) $NaIO_3 + NaI$
(c) $NaOI + NaIO_3 + NaI$
(d) $NaIO_4 + NaI$

66. A drug effective in the treatment of pneumonia, bronchitis, etc., is
(a) streptomycin (b) chloramphenicol
(c) penicillin (d) sulphaguanidine.

67. Which step is not involved in hydrometallurgical process?
(a) $Cu_2S + 2Cu_2O \rightarrow 6Cu + SO_2$
(b) $CuFeS_2 + 2H_2SO_4 \rightarrow CuSO_4 + FeSO_4 + 2H_2S$
(c) $CuSO_4 + Fe \rightarrow FeSO_4 + Cu$
(d) $CuCO_3 + H_2SO_4 \rightarrow CuSO_4 + H_2O + CO_2$

68. A current of 2 ampere passing for 5 hours through a molten tin salt deposits 22.2 g of tin. What is the oxidation state of tin in the salt?
(a) 1 (b) 2 (c) 3 (d) 4

69. An organic amino compound reacts with aqueous nitrous acid at low temperature to produce an oily nitrosoamine. The compound is
(a) CH_3NH_2 (b) $CH_3CH_2NH_2$
(c) $(C_2H_5)_2NH$ (d) $(C_2H_5)_3N$

70. What is the equation form of Langmuir adsorption isotherm under high pressure?
(a) $\frac{x}{m} = \frac{a}{b}$ (b) $\frac{x}{m} = aP$
(c) $\frac{x}{m} = \frac{1}{aP}$ (d) $\frac{x}{m} = \frac{b}{a}$

71. What mass of $BaSO_4$ will dissolve in 450 mL aqueous solution? [K_{sp} for $BaSO_4 = 1.0 \times 10^{-10}$]
(a) 0·01 g (b) 0.23 g
(c) 0.001 g (d) 2.3×10^{-3} g

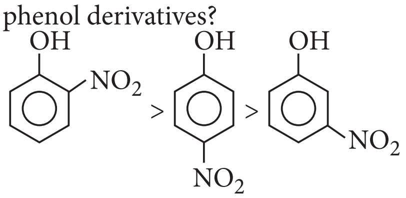
72. Bleaching powder loses its power on keeping for a long time because
(a) it changes into calcium hypochlorate
(b) it changes into $CaCl_2$ and $Ca(OH)_2$
(c) it absorbs moisture
(d) it changes into calcium chloride and calcium chlorate.

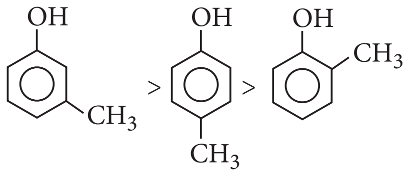
73. All the alkali metals give characteristic flame test. The decreasing order of the frequency of light emitted by them is
(a) Li > Na > K > Rb > Cs
(b) Li > Na = K = Rb > Cs
(c) Li = Na > K > Rb = Cs
(d) Cs > Rb > K > Na > Li

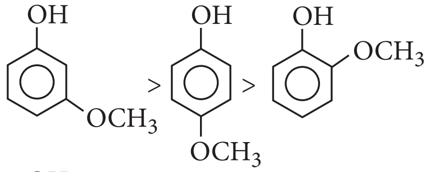
74. Which of the following amino acid has two –COOH groups?
(a) Histidine (b) Aspartic acid
(c) Lysine (d) Valine

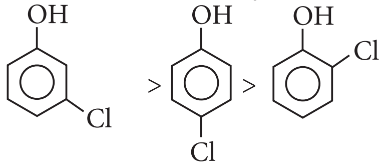
75. The number of S—S bonds in sulphur trioxide trimer (S_3O_9) is
(a) three (b) two (c) one (d) zero.

76. Which of the following is the correct acidity order for phenol derivatives?

(a) 

(b) 

(c) 

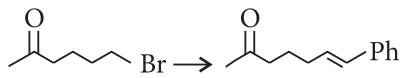
(d) 

77. Nitrous oxide decomposes into N_2 and O_2, where reactants and products are in gas phase. If the reaction is first order then the rate constant for this reaction in terms of pressure, *i.e.*, P_i = initial pressure, P_f = final pressure may be denoted as

(a) $k=\frac{1}{t}\ln\frac{P_i}{P_i-P_f}$ (b) $k=\frac{1}{t}\ln\frac{P_i}{P_f}$

(c) $k=\frac{1}{t}\ln\frac{P_i}{3P_i-2P_f}$ (d) $k=\frac{1}{t}\ln\frac{P_i}{2P_f-3P_i}$

78. Which of the following combination of reagents can bring the given transformation?



(a) Mg/ether, PhCHO/Δ, $(CH_2OH)_2/H^+$, H_3O^+
(b) $(CH_2OH)_2/H^+$, Mg/ether, PhCHO/Δ, H_3O^+
(c) Alc. KOH, PhCHO/Δ, Mg/ether, H_3O^+
(d) $(CH_2OH)_2/H^+$, PhCHO/Δ, H_3O^+

79. During the transformation of ${}^{a}_{c}X$ to ${}^{b}_{d}Y$ by α and β-decay, the number of β-particles emitted are

(a) $\frac{a-b}{4}$ (b) $d+\frac{a-b}{2}+c$

(c) $d+\left(\frac{a-b}{2}\right)-c$ (d) $2c-d+a-b$

80. Which of the following sets of species does not follow octet rule?
(a) CO, PCl_5, PCl_3, $AlCl_3$
(b) CO, B_2H_6, NH_3, H_2O
(c) $AlCl_3$, BF_3, PCl_5, SF_6
(d) H_2O, NH_3, CO_2, $AlCl_3$

MATHEMATICS

81. The complex number z, satisfying the condition $\arg\left(\frac{z-1}{z+1}\right)=\frac{\pi}{4}$ lies on a
(a) circle (b) line
(c) x-axis (d) y-axis

82. If $x > 0$, then solution of $\left|x+\frac{1}{x}\right|<4$ is
(a) $-2-\sqrt{3}<x<-2+\sqrt{3}$
(b) $2-\sqrt{3}<x<7+\sqrt{3}$
(c) $2-\sqrt{3}<x<2+\sqrt{3}$
(d) none of these

83. If $x = \log_5 3 + \log_7 5 + \log_9 7$, then
(a) $x\ge\frac{3}{\sqrt[3]{2}}$ (b) $x\ge\frac{5}{\sqrt[3]{3}}$
(c) $x\ge\frac{3}{\sqrt[3]{5}}$ (d) none of these

84. $\sum_{r=1}^{n}\left(\sum_{p=0}^{r-1}{}^{n}C_r\cdot{}^{r}C_p\cdot 2^p\right)$ is equal to
(a) 4^n-3^n+1 (b) 4^n-3^n-1
(c) 4^n-3^n+2 (d) 4^n-3^n

85. If the vectors $a\hat{i}+\hat{j}+\hat{k}, \hat{i}+b\hat{j}+\hat{k}$ and $\hat{i}+\hat{j}+c\hat{k}$ are coplanar, then the value of $\frac{1}{1-a}+\frac{1}{1-b}+\frac{1}{1-c}$ is
(a) 0 (b) 1 (c) 2 (d) 3

86. C_1 and C_2 are two circles of unit radius with centres at (0, 0) and (1, 0) respectively. C_3 is a circle of unit radius, passes through the centres of the circles C_1 and C_2 and have its centre above x-axis. Equation of the common tangent to C_1 and C_3 which does not pass through C_2, is
(a) $x-\sqrt{3}y+2=0$ (b) $\sqrt{3}x-y+2=0$
(c) $\sqrt{3}x-y-2=0$ (d) $x+\sqrt{3}y+2=0$

87. If the function $f:[1,\infty)\to[1,\infty)$ is defined by $f(x)=2^{x(x-1)}$, then $f^{-1}(y)$ is equal to
(a) $\left(\frac{1}{2}\right)^{x(x-1)}$ (b) $\frac{1}{2}(1+\sqrt{1+4\log_2 y})$
(c) $\frac{1}{2}(1-\sqrt{1-4\log_2 x})$ (d) not defined

88. If 3 $\sin^2\theta$ + 2 $\sin^2\phi$ = 1 and 3 $\sin 2\theta$ = 2 $\sin 2\phi$, $0<\theta<\frac{\pi}{2}$ and $0<\ \phi<\frac{\pi}{2}$, then the value of $\theta + 2\phi$ is

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$
(c) π (d) none of these

89. The inclination of the straight line passing through the point (–3, 6) and the mid-point of the line joining the points(4, –5) and (–2, 9) is

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{3}$ (d) $\frac{3\pi}{4}$

90. If $\omega(\neq 1)$ is a cube root of unity and $(1+\omega)^7 = A + B\omega$, then A and B respectively are

(a) 0, 1 (b) 1, 1 (c) 1, 0 (d) –1, 1

91. A man has ₹ 1500 for purchase of rice and wheat. A bag of rice and a bag of wheat cost ₹ 180 and ₹ 120 respectively. He has a storage capacity of 10 bags only. He earns a profit of ₹ 11 on each rice bag and ₹ 9 on each wheat bag. Find the maximum profit.

(a) ₹ 110 (b) ₹ 90 (c) ₹ 95 (d) ₹ 100

92. An ellipse slides between two lines at right angle to one another. Then, the locus of its centre is a/an

(a) circle (b) parabola
(c) ellipse (d) hyperbola

93. If $\sin^{-1}a + \sin^{-1}b + \sin^{-1}c = \pi$, then the value of $a\sqrt{(1-a^2)}+b\sqrt{(1-b^2)}+c\sqrt{(1-c^2)}$ will be

(a) $2abc$ (b) abc (c) $\frac{1}{2}abc$ (d) $\frac{1}{3}abc$

94. If $A=\begin{bmatrix}6 & 8 & 5\\ 4 & 2 & 3\\ 9 & 7 & 1\end{bmatrix}$ is the sum of a symmetric matrix B and skew-symmetric matrix C, then B is

(a) $\begin{bmatrix}6 & 6 & 7\\ 6 & 2 & 5\\ 7 & 5 & 1\end{bmatrix}$ (b) $\begin{bmatrix}0 & 2 & -2\\ -2 & 5 & -2\\ 2 & 2 & 0\end{bmatrix}$

(c) $\begin{bmatrix}6 & 6 & 7\\ -6 & 2 & -5\\ -7 & 5 & 1\end{bmatrix}$ (d) $\begin{bmatrix}0 & 6 & -2\\ 2 & 0 & -2\\ -2 & -2 & 0\end{bmatrix}$

95. The system $y^{(x^2+7x+12)} = 1$ and $x + y = 6, y > 0$ has

(a) no solution (b) one solution
(c) two solutions (d) more than 2 solutions

96. The differential equation of the family of parabolas with focus at the origin and the x-axis as axis, is

(a) $y\left(\frac{dy}{dx}\right)^2+4x\frac{dy}{dx}=4y$

(b) $y\left(\frac{dy}{dx}\right)^2+2x\frac{dy}{dx}=y$

(c) $y\left(\frac{dy}{dx}\right)^2+y=2x\frac{dy}{dx}$

(d) $y\left(\frac{dy}{dx}\right)^2+2xy\frac{dy}{dx}+y=0$

97. Equation of the line passing through the point of intersection of lines $2x - 3y + 4 = 0$, $3x + 4y - 5 = 0$ and perpendicular to line $6x - 7y + 3 = 0$, is

(a) $119x + 102y + 125 = 0$
(b) $119x + 102y = 125$
(c) $119x - 102y = 125$
(d) none of these

98. If $\cos^{-1}\alpha + \cos^{-1}\beta + \cos^{-1}\gamma = 3\pi$, then $\alpha(\beta + \gamma) + \beta(\gamma + \alpha) + \gamma(\alpha + \beta)$ is equal to

(a) 0 (b) 1 (c) 6 (d) 12

99. If $x_1, x_2, \ldots, x_{50}$ are fifty real numbers such that $x_r < x_{r+1}$ for $r = 1, 2, 3,\ldots, 49$. Five numbers out of these are picked up at random. The probability that the five numbers have x_{20} as the middle number, is

(a) $\frac{^{20}C_2\times{}^{30}C_2}{^{50}C_5}$ (b) $\frac{^{30}C_2\times{}^{19}C_2}{^{50}C_5}$

(c) $\frac{^{19}C_2\times{}^{31}C_2}{^{50}C_5}$ (d) none of these

100. Matrix A is such that $A^2 = 2A - I$, where I is the identity matrix. Then, for $n \geq 2$, A^n is equal to

(a) $nA - (n - 1)I$ (b) $nA - I$
(c) $2^{n-1}A - (n - 1)I$ (d) $2^{n-1}A - I$

101. The product of the lengths of perpendiculars drawn from any point on the hyperbola $x^2 - 2y^2 - 2 = 0$ to its asymptotes, is

(a) $\frac{1}{2}$ (b) $\frac{2}{3}$ (c) $\frac{3}{2}$ (d) 2

102. If $f(x)=\frac{x+2}{2x+3}$. Then, $\int\left\{\frac{f(x)}{x^2}\right\}^{1/2}dx$ = $\frac{1}{\sqrt{2}}g\left(\frac{1+\sqrt{2f(x)}}{\sqrt{2f(x)}-1}\right)-\sqrt{\frac{2}{3}}h\left(\frac{\sqrt{3f(x)}+\sqrt{2}}{\sqrt{3f(x)}-\sqrt{2}}\right)+C$, where

(a) $g(x) = \tan^{-1}x$, $h(x) = \log|x|$
(b) $g(x) = \log|x|$, $h(x) = \tan^{-1}x$
(c) $g(x) = h(x) = \tan^{-1}x$
(d) $g(x) = \log|x|$, $h(x) = \log|x|$

103. The fuel charges for running a train are proportional to the square of the speed generated in mile/h and costs ₹ 48 per h at 16 miles/h. The most economical speed if the fixed charges *i.e.*, salaries etc. amount to ₹ 300 per h is

(a) 10 mile/h (b) 20 mile/h
(c) 30 mile/h (d) 40 mile/h

104. If two numbers p and q are chosen randomly from the set {1, 2, 3, 4} with replacement, then the probability that $p^2 \geq 4q$ is equal to

(a) $\frac{1}{4}$ (b) $\frac{3}{16}$ (c) $\frac{1}{2}$ (d) $\frac{7}{16}$

105. Let A and B be two points with position vectors $\vec{a}$ and $\vec{b}$ with respect to the origin O. If the point C on OA is such that $2\overrightarrow{AC} = \overrightarrow{CO}, \overrightarrow{CD}$ is parallel to $\overrightarrow{OB}$ and $|\overrightarrow{CD}| = 3|\overrightarrow{OB}|$, then $\overrightarrow{AD}$ is

(a) $3\vec{b} - \frac{1}{3}\vec{a}$ (b) $3\vec{a}$
(c) $\frac{1}{3}\vec{b}$ (d) $3\vec{b} + \frac{1}{3}\vec{a}$

106. Let $f : R \to R$ such that $f(x + 2y) = f(x) + f(2y) + 4xy$ $\forall\ x, y \in R$ and $f'(0) = 0$. If $I_1 = \int_0^1 f(x)dx$, $I_2 = \int_{-1}^0 f(x)dx$ and $I_3 = \int_{1/2}^2 f(x)dx$, then

(a) $I_1 = I_2 > I_3$ (b) $I_1 > I_2 > I_3$
(c) $I_1 = I_2 < I_3$ (d) $I_1 < I_2 < I_3$

107. The area bounded by the curve $y = x^4 - 2x^3 + x^2 + 3$, the axis of abscissae and two ordinates corresponding to the points of minimum of function $y(x)$ is

(a) $\frac{10}{3}$ sq. units (b) $\frac{27}{10}$ sq. units
(c) $\frac{21}{10}$ sq. units (d) none of these

108. If a_r is the coefficient of x^r, in the expansion of $(1 + x + x^2)^n$, then $a_1 - 2a_2 + 3a_3 - ... - 2n\ a_{2n}$ is equal to

(a) 0 (b) n (c) $-n$ (d) $2n$

109. If $a < b < c < d$, then the roots of the equation $(x - a)(x - c) + 2(x - b)(x - d) = 0$ are

(a) real and distinct (b) real and equal
(c) imaginary (d) none of these

110. Let a_n be the n^{th} term of the G.P. of positive numbers. Let $\sum_{n=1}^{100} a_{2n} = \alpha$ and $\sum_{n=1}^{100} a_{2n-1} = \beta$, such that $\alpha \neq \beta$, then the common ratio is

(a) $\frac{\alpha}{\beta}$ (b) $\frac{\beta}{\alpha}$ (c) $\sqrt{\left(\frac{\alpha}{\beta}\right)}$ (d) $\sqrt{\left(\frac{\beta}{\alpha}\right)}$

111. If C_r stands for ${}^nC_r$ and $\frac{C_0}{1} - \frac{C_1}{3} + \frac{C_2}{5} - ...$ $... + \frac{(-1)^n C_n}{2n+1} = k\int_0^1 x(1-x^2)^{n-1}dx$, then k is equal to

(a) $\frac{2^{2n+1} \cdot n \cdot n!}{(2n+1)!}$ (b) $2^{2n} \cdot \frac{n!}{(2n+1)!}$
(c) ${}^{2n+1}C_n$ (d) $\frac{2^{2n+1} \cdot n \cdot (n!)^2}{(2n+1)!}$

112. The equation $3 \sin^2 x + 10 \cos x - 6 = 0$ is satisfied, if

(a) $x = n\pi \pm \cos^{-1}\left(\frac{1}{3}\right)$ (b) $x = 2n\pi \pm \cos^{-1}\left(\frac{1}{3}\right)$
(c) $x = n\pi \pm \cos^{-1}\left(\frac{1}{6}\right)$ (d) $x = 2n\pi \pm \cos^{-1}\left(\frac{1}{6}\right)$

113. The number of all possible triplets (x, y, z) such that $(x + y) + (y + 2z) \cos 2\theta + (z - x)\sin^2\theta = 0$ for all θ is

(a) 0 (b) 1 (c) 3 (d) infinite

114. Solution set of the inequality $\sqrt{x} - 3 \leq \frac{2}{\sqrt{x} - 2}$ is

(a) $2 \leq x \leq 3$ (b) [0, 1] $\cup$ [2, 16]
(c) [0, 1] $\cup$[3, 16] (d) none of these

115. The coordinates of the point on the parabola $y^2 = 8x$, which is at minimum distance from the circle $x^2 + (y + 6)^2 = 1$, are

(a) (2, –4) (b) (18, – 12)
(c) (2, 4) (d) none of these

116. Let $\vec{b} = 4\hat{i} + 3\hat{j}$ and $\vec{c}$ be two vectors perpendicular to each other in the xy-plane. All vectors in the same plane having projections 1 and 2 along $\vec{b}$ and $\vec{c}$ respectively, are given by

(a) $2\hat{i} - \hat{j}, -\frac{2}{5}\hat{i} + \frac{11}{5}\hat{j}$ (b) $2\hat{i} + \hat{j}, \hat{i} + \hat{j}$
(c) $\hat{i} - \hat{j}, \hat{i} + \hat{j} + \hat{k}$ (d) $\hat{i} + \hat{j} + \hat{k}, \hat{i} - \hat{k}$

117. Given the points $A(0, a)$ and $B(0, -a)$, the equation of the locus of point $P(x, y)$ such that $|AP - BP| = 6$ is

(a) $\frac{x^2}{a^2 - 9} - \frac{y^2}{9} = 1$ (b) $\frac{x^2}{a^2 - 9} - \frac{y^2}{9} + 1 = 0$
(c) $\frac{x^2}{9} - \frac{y^2}{a^2 - 9} = 1$ (d) $\frac{x^2}{9} - \frac{y^2}{a^2 - 9} + 1 = 0$

118. If $z_r = \cos\dfrac{r\alpha}{n^2} + i\sin\dfrac{r\alpha}{n^2}$, where $r = 1, 2, 3, \ldots\ldots, n$, then $\lim\limits_{n\to\infty} z_1 z_2 z_3 \ldots\ldots z_n$ is equal to

(a) $\cos\alpha + i\sin\alpha$ (b) $\cos\left(\dfrac{\alpha}{2}\right) - i\sin\left(\dfrac{\alpha}{2}\right)$

(c) $e^{i\alpha/2}$ (d) $\sqrt[3]{e^{i\alpha}}$

119. If the lines $2x + 3y + 1 = 0$ and $3x - y - 4 = 0$ lie along diameters of a circle of circumference 10π, then the equation of the circle is

(a) $x^2 + y^2 - 2x + 2y - 23 = 0$
(b) $x^2 + y^2 - 2x - 2y - 23 = 0$
(c) $x^2 + y^2 + 2x + 2y - 23 = 0$
(d) $x^2 + y^2 + 2x - 2y - 23 = 0$

120. The arbitrary constant on which the value of

$$\Delta = \begin{vmatrix} 1 & \alpha & \alpha^2 \\ \cos(p-d)a & \cos pa & \cos(p-d)a \\ \sin(p-d)a & \sin pa & \sin(p-d)a \end{vmatrix}$$

does not depend, is

(a) α (b) p (c) d (d) a

121. Domain of $f(x) = \sin^{-1}[2 - 4x^2]$([.] denotes the greatest integer function) is

(a) $[-1, 1]$ (b) $(-2, 2)$

(c) $\left[-\dfrac{\sqrt{3}}{2}, 0\right) \cup \left(0, \dfrac{\sqrt{3}}{2}\right]$ (d) none of these

122. The order of the differential equation whose general solution is given by $y = (c_1 + c_2)\cos(x + c_3) - c_4 e^{x+c_5}$, where c_1, c_2, c_3, c_4 and c_5 are arbitrary constants is

(a) 5 (b) 6 (c) 3 (d) 2

123. The angle between the planes whose vector equations are $\vec{r}\cdot(2\hat{i} + 2\hat{j} - 3\hat{k}) = 5$ and $\vec{r}\cdot(3\hat{i} - 3\hat{j} + 5\hat{k}) = 3$ is

(a) $\cos^{-1}\left(\dfrac{15}{\sqrt{731}}\right)$ (b) $\cos^{-1}\left(\dfrac{15}{\sqrt{7}}\right)$

(c) $\dfrac{\pi}{4}$ (d) $\dfrac{\pi}{2}$

124. In a random experiment, the success is thrice that of failure. If the experiment is repeated 5 times, then the probability that atleast 4 times favourable is

(a) $\dfrac{1296}{2048}$ (b) $\dfrac{1003}{2048}$

(c) $\dfrac{1203}{2048}$ (d) none of these

125. Evaluate : $\displaystyle\int \frac{\tan\theta + \tan^3\theta}{1 + \tan^3\theta}\, d\theta$

(a) $-\dfrac{1}{3}\log|1+\tan\theta| - \dfrac{1}{6}\log|\tan^2\theta - \tan\theta + 1| - \dfrac{1}{\sqrt{3}}\tan^{-1}\left(\dfrac{2\tan\theta - 1}{\sqrt{3}}\right) + C$

(b) $-\dfrac{1}{3}\log|1+\tan\theta| + \dfrac{1}{6}\log|\tan^2\theta - \tan\theta + 1| + \dfrac{1}{\sqrt{3}}\tan^{-1}\left(\dfrac{2\tan\theta - 1}{\sqrt{3}}\right) + C$

(c) $-\dfrac{1}{3}\log|1+\tan\theta| + \dfrac{1}{6}\log|\tan^2\theta + \tan\theta + 1| - \dfrac{1}{\sqrt{3}}\tan^{-1}\left(\dfrac{2\tan\theta + 1}{\sqrt{3}}\right) + C$

(d) none of these

LOGICAL REASONING

126. There is a certain relationship between the pair of words on the either side of : :. Identify the relationship and find the missing word.

Genuine : Authentic : : Mirage : ?

(a) Reflection (b) Hideout
(c) Illusion (d) Image

127. Complete the given series.

1, 5, 14, 30, 55, …

(a) 97 (b) 95 (c) 91 (d) 55

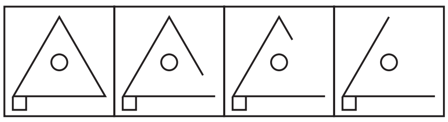
128. Find the missing number, if certain rule is followed row-wise or column-wise.

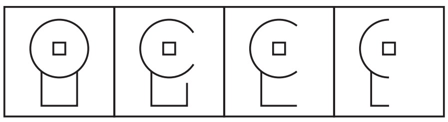
5	9	8	7
8	6	9	10
7	13	?	19
5	7	8	9

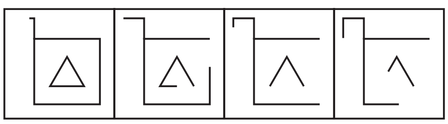
(a) 9 (b) 10
(c) 12 (d) 15

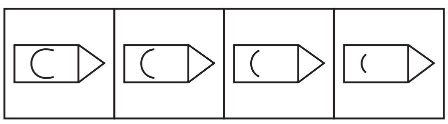
129. Which one of the given sets of figures follows the following rule?

"Closed figures become more and more open and open figures become more and more closed."

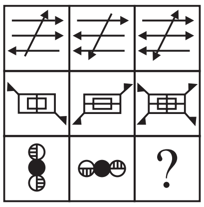
(a) 

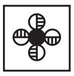
(b) 

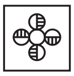
(c) 

(d) 

130. Which of the following options completes the figure matrix ?



(a) 

(b) 

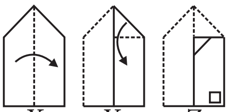
(c) 

(d) none of these

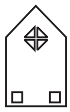
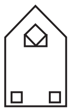
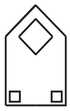
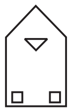
131. Three of the following four have similar relationships and hence form a group. Which one does not belong to the gorup?

(a) SAFETY : VYICWW
(b) SMOKER : VKRIHP
(c) SERIES : VCUGHQ
(d) HEALTH : KCYJYF

132. Consider the following three figures, marked X, Y, Z showing one fold in X, another fold in Y and cut in Z. From amongst the answer figures (a), (b), (c), and (d), select the one showing the unfolded position of Z.



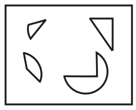
X Y Z

(a)  (b)  (c)  (d) 

133. In the given letter sequence, some letters are missing which are given in that order as one of the four alternatives under it.
Find out the correct option.
ab — d — aaba — na — badna — b

(a) andaa (b) babda
(c) badna (d) dbanb

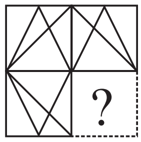
134. In the following question, find out which of the figures (a), (b), (c) and (d) can be formed from the pieces given in figure.

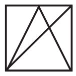
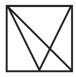
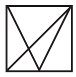


(X)

(a)  (b)  (c)  (d) 

135. Select a figure from the options which when placed in the blank space of the given figure would complete the pattern.



(a)  (b)  (c)  (d) 

ENGLISH

136. Complete the sentence.
The telephone .... several times before I answered it.

(a) has rung (b) was ringing
(c) would ring (d) had rung

137. Choose the correct synonym.
Audacious

(a) Manifest (b) Venture
(c) Obvious (d) Daring

138. Choose the correct antonym.
Boisterous

(a) Good (b) Happy
(c) Calm (d) Comfortable

139. Choose the one alternative which can be substituted for the given description.
One who has suddenly gained new wealth, power or prestige-

(a) Maverick (b) Parvenu
(c) Aristocrat (d) Affluent

140. Choose the correct spelling of the given words.

(a) Sacriligious (b) Sacreligious
(c) Sacrilegeous (d) Sacrilegious

141. Choose the part of the sentence that has an error.

(a) One of them (b) forget to take
(c) their mobile phone (d) from the office.

Direction (142-145): Read the passage and answer the questions that follow.

PASSAGE

It is to progress in the human sciences that we must look to undo the evils which have resulted from a knowledge of the physical world hastily and superficially acquired by populations unconscious of the changes in themselves that the new knowledge has made imperative. The road to a happier world than any known in the past lies open before us if atavistic destructive passions can be kept in leash while the necessary adaptations are made. Fears are inevitable in our time, but hopes are equally rational and far more likely to bear good fruit. We must learn to think rather

less of the dangers to be avoided than of the good that will lie within our grasp if we can believe in it and let it dominate our thoughts. Science, whatever unpleasant consequences it may have by the way, is in its very nature a liberator, a liberator of bondage to physical nature and is to come, a liberator from the weight of destructive passions. We are on the threshold of utter disaster or unprecedentedly glorious achievement. No previous age has been fraught with problems so momentous; and it is to science that we must look to for a happy future.

142. What does science liberate us from?
It liberates us from________.
(a) idealistic hopes of a glorious future
(b) slavery to physical nature and from passions
(c) bondage to physical nature
(d) fears and destructive passions

143. To carve out a bright future a man should ______.
(a) cultivate a positive outlook
(b) analyse dangers that lie ahead
(c) try to avoid dangers
(d) overcome fears and dangers

144. If man's bestial yearning is controlled, ________.
(a) the future will be brighter than the present
(b) the future will be tolerant
(c) the present will be brighter than the future
(d) the present will become tolerant

145. Fears and hopes, according to the author _______.
(a) are irrational
(b) are closely linked with the life of modern man
(c) can yield good results
(d) can bear fruit

Direction (146-150): Choose the correct alternative to fill in the blanks to make a meaningful sentence.

146. Freedom and equality are the .... rights of every human being.
(a) incalculable (b) institutional
(c) inalienable (d) inscrutable

147. This article tries to .... us with problems of poor nations so that we help them more effectively.
(a) convince (b) project
(c) allow (d) acquaint

148. Eight scientists have .... the national awards for outstanding contribution and dedication to the profession.
(a) bagged (b) conferred
(c) bestowed (d) picked

149. Ravi had to drop his plan of going to the picnic as he had certain .... to meet during that period.
(a) urgencies (b) commitments
(c) preparations (d) observations

150. The speaker did not properly use the time as he went on .... on one point alone.
(a) deliberating (b) diluting
(c) dilating (d) devoting

ANSWER KEYS

1.	(a)	2.	(a)	3.	(a)	4.	(c)	5.	(c)
6.	(a)	7.	(c)	8.	(d)	9.	(d)	10.	(b)
11.	(c)	12.	(c)	13.	(c)	14.	(b)	15.	(c)
16.	(d)	17.	(c)	18.	(d)	19.	(b)	20.	(c)
21.	(a)	22.	(c)	23.	(b)	24.	(b)	25.	(a)
26.	(c)	27.	(c)	28.	(b)	29.	(b)	30.	(c)
31.	(b)	32.	(a)	33.	(d)	34.	(a)	35.	(a)
36.	(d)	37.	(a)	38.	(c)	39.	(c)	40.	(a)
41.	(b)	42.	(b)	43.	(b)	44.	(a)	45.	(b)
46.	(a)	47.	(b)	48.	(a)	49.	(c)	50.	(c)
51.	(c)	52.	(b)	53.	(b)	54.	(a)	55.	(a)
56.	(a)	57.	(a)	58.	(c)	59.	(b)	60.	(c)
61.	(c)	62.	(a)	63.	(d)	64.	(a)	65.	(b)
66.	(c)	67.	(a)	68.	(b)	69.	(c)	70.	(a)
71.	(c)	72.	(d)	73.	(d)	74.	(b)	75.	(d)
76.	(b)	77.	(c)	78.	(b)	79.	(c)	80.	(c)
81.	(a)	82.	(c)	83.	(a)	84.	(d)	85.	(b)
86.	(b)	87.	(b)	88.	(b)	89.	(d)	90.	(b)
91.	(d)	92.	(a)	93.	(a)	94.	(a)	95.	(d)
96.	(b)	97.	(b)	98.	(c)	99.	(b)	100.	(a)
101.	(b)	102.	(d)	103.	(d)	104.	(d)	105.	(a)
106.	(c)	107.	(d)	108.	(c)	109.	(a)	110.	(a)
111.	(d)	112.	(b)	113.	(d)	114.	(d)	115.	(a)
116.	(a)	117.	(b)	118.	(c)	119.	(a)	120.	(b)
121.	(c)	122.	(c)	123.	(a)	124.	(a)	125.	(b)
126.	(c)	127.	(c)	128.	(d)	129.	(c)	130.	(c)
131.	(d)	132.	(c)	133.	(a)	134.	(c)	135.	(d)
136.	(d)	137.	(d)	138.	(c)	139.	(b)	140.	(d)
141.	(c)	142.	(b)	143.	(a)	144.	(a)	145.	(b)
146.	(c)	147.	(d)	148.	(a)	149.	(b)	150.	(c)

6 months, over a dozen experts: making of a CBSE question paper

Monday's Class XII maths paper has been criticised in schools, homes and Parliament. Indian Express explains the process by which CBSE question papers are set — a secret, complex exercise with strict do's & dont's.

The Central Board of Secondary Education (CBSE) announced on Wednesday that it would place before a panel of experts the feedback received from teachers, students and examiners on the Class XII Mathematics paper held on Monday, and "take remedial measures before evaluation".

This followed uproar over the "extraordinarily lengthy and tough" maths paper; specifically, students and teachers complained that some 1-mark questions took disproportionately long to answer. Union Urban Development Minister M Venkaiah Naidu told Parliament that "certain questions were very tough and even bright students couldn't answer them effectively".

This is the second year in a row that the maths paper has been criticised — raising questions over the process by which CBSE designs question papers. According to Board officials, the system, governed by the CBSE's examination bylaws, contains elaborate and multiple levels of monitoring before, during and after the students take the exam.

Before the exam

The CBSE's guidelines for curriculum lay down the number of periods to be allotted to each topic in class, and weightage in the question paper. For Class XII maths, calculus has been allotted 80 periods and 44 marks — the most. Algebra has 50 periods but only 13 marks; vectors and 3D geometry, 30 periods and 17 marks.

In 2014, the guidelines redefined the "typology" and "design" of the question paper. In maths, application-based questions, which "use abstract information in concrete situation, to apply knowledge to new situations", should get 29% marks, it said. The High Order Thinking Skills (HOTS) questions, which test "analysis and synthesis — classify, compare, contrast, or differentiate between different pieces of information, organise and/or integrate unique pieces of information from a variety of sources", were allotted 15%. "Remembering" or recall-based questions got 20%, and "understanding" or comprehension of concepts, 22%.

Paper-setting, Stage I

Setting the question paper for each subject takes six months or more. It is prepared by the efforts of 9-17 people with specific qualifications for the job. They come from all over the country, and none of them knows the identity of the others.

The process begins in August-September, when Class XII students are still finishing their first term exams. For each subject, the Board appoints 4-7 paper "setters", each of whom has a postgraduate degree in the concerned subject or an allied subject, at least 10 years' experience teaching the concerned subject at secondary or senior secondary level, or is employed at a state- or national-level education agency set up by the government. According to the rules, the paper setters should not have "written or revised a guide-book, help-book, key or similar other matter, with whatsoever name, relating to the subject". They should also not have given private tuition that year, and no member of their immediate family should be appearing in that year's exams.

The setters are given question papers of previous years, and the curriculum design and structure of the papers — their "typology" and "design". Following these guidelines, a setter prepares a question paper in about two months on average, and sends it to CBSE.

Paper-setting, Stage II

A second set of experts enter the picture now. These "moderators" check if laid-down standards have been followed on the syllabus, difficulty level, and length of the paper. About 5-10 moderators are appointed for each subject every year. They have the same qualifications as setters, but are a different set of individuals.

"Since students from different socio-economic backgrounds and intellectual levels take the paper, this is an effort to ensure there is some parity. This is our second level of monitoring," said a senior official.

According to the bylaws, the moderators should ensure that "each question paper has been set according to the syllabus of the subject, blue print, design and text books/recommended books", and complies with the unit-wise weightage given in a subject's curriculum. The bylaws state that variations of marks, if any, under different sub-units of the subject should be kept "at the minimum".

The moderators also prepare a marking scheme, which includes expected answers, distribution of marks, and the marks to be allotted for every step of the solution. They have to mention against each question the "approximate time" needed to solve it by an "average student who has carefully studied the course and has prepared for the examination methodically". The moderators must ensure no question is erroneously or ambiguously worded, which could lead to "an interpretation different from the question intends to convey".

After the exam

Subject experts from across the country meet after the examination that same day to consider feedback received during and after the exam. For this year's controversial maths paper, 17 senior teachers from all over India met. For English (Core), 11 subject experts were identified, 15 for Physics, 17 each for Biology and Chemistry. Based on examination-day feedback, the experts revise the marking scheme. "They define the marks to be allotted for every step of every question afresh. This is given to our evaluators afresh, with modifications, if any, to the marking scheme defined by paper setters and moderators," the official explained.

Evaluators can't see the correct roll numbers of examinees. For each of the 10 regions under CBSE, a Professor or Principal of a college, or a Reader or Senior Lecturer is appointed Chief Secrecy Officer, who then notifies a team — made of individuals who are at least college lecturers — to put fictitious roll numbers on every answer sheet. Again, no immediate relative of a secrecy officer should be appearing for the exam that year.

A Principal, Vice-Principal, or Post Graduate Teacher of an affiliated school is appointed Head Examiner for every subject. The Head Examiner has to monitor the evaluation to ensure uniform evaluation principles are being followed by all evaluators.

Courtesy : The Indian Express

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